Math 131 Homework: Day 5

My Office Hours: M & W 12:30–2:00, Tu 2:30–4:00, & F 1:15-2:30 or by appointment. **Math Intern** Sun: 12–6pm; M 3-10pm; Tu 2–6, 7–1pm; W and Th: 5-10 pm in Lansing 310. Website: `http://math.hws.edu/~mitchell/Math131S13/index.html`.

☕ Practice

Review Lab 2 and the answers online.

1. ℹ Review Section 5.2 which finishes our introduction to the Riemann Integral giving a number of its basic properties. Begin to read Section 5.3 on the **Fundamental Theorem of Calculus**. Any theorem with this name must be important.

- **a)** Understand the definition of a **Riemann Sum** (p. 323).
- **b)** Memorize the definition of a **Definite Integral** (p. 324).
- **c)** Understand Theorem 5.2 (p. 325).
- **d)** Remember, geometrically, a definite integral $\int_a^b f(x)\,dx$ is the net area 'under' f on $[a, b]$. We have 'solved' the area problem.
- **e)** Review the properties of Definite Integrals on pages 327–329. These are summarized on page 329.

2.
- **a)** Try page 331–332 #5, 7, 19, 21, 23–29 odd.
- **b)** These next problems are easy but use important concepts: Page 332–333 #31, 33, 39, 41, 43.
- **c)** Here are some additional Riemann Sums to practice: Page 333 #47 and 49.

Math 131 Hand In Day 5

0. Do WeBWorK set Day05. (Due Tuesday but some problems may help you with the homework below.)

1. Use an appropriate Riemann sum to evaluate $\int_2^3 (x^2 - 4)\,dx$. Show all your work. (Compare to Lab 2, problem 5).

2. Let $f(x) = x^2 + x$ on $[0, 2]$.

- **a)** Explain why Upper(n) = Right(n) and Lower(n) = Left(n).
- **b)** Determine Upper(n). Show the details.
- **c)** Determine $\int_0^2 (x^2 + x)\,dx$.
- **d)** **Extra Credit:** Show that Lower(n) = Upper$(n) - \frac{16}{n}$. Comment: Written this way makes it easy to see why $\lim_{n\to\infty} \text{Upper}(n) = \lim_{n\to\infty} \text{Lower}(n)$.

3. Use geometry (not Riemann sums) to determine the following definite integrals. Hint: See Example 3 in Section 5.2. Sketch a graph of the function to illustrate the region.

a) $\displaystyle\int_0^2 (6 - 3x)\,dx$ **b)** $\displaystyle\int_{-2}^1 -|x|\,dx$ **c)** $\displaystyle\int_{-3}^3 \sqrt{9 - x^2}\,dx$ **d)** $\displaystyle\int_0^4 f(x)\,dx$, where $f(x) = \begin{cases} 3 & \text{if } x \le 2 \\ x & \text{if } x > 2. \end{cases}$

4. Using properties of the integral:

- **a)** Page 332 #36 and 38. Show how you obtained your answer.
- **b)** Page 332 #40(a,c). Use properties of the integral. Show your 'work.'
- **c)** Page 332 #44. Use properties of the integral. Note the order of the limits.

5. Compare to Lab 2, Problem 7. Review it and the answers on line if needed. Make the 'adjustments' necessary (see the lab) to determine the following antiderivatives. (Check your answers by taking the derivative.)

a) $\displaystyle\int \sqrt{7x}\,dx$ **b)** $\displaystyle\int \sin\frac{\pi x}{3}\,dx$

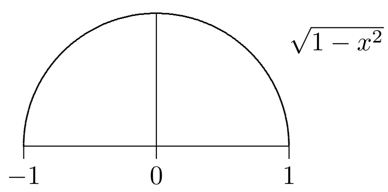
6. This problem uses the online applet `xFunctions`. There's a link to this applet at the course webpage or use `http://math.hws.edu/~mitchell/Math131S13/Math131/xFunctions.html` See the image below. ☞ NOTE: If you have problems, the program should work from the Colleges' networked computers. Macintosh users may encounter difficulties. Try using the Firefox browser.

a) What is the area of a semi-circle of radius 1? Give your answer in terms of π and also as a decimal rounded correctly to four places.

b) The equation of the semi-circle of radius 1 centered at the origin is $f(x) = \sqrt{1-x^2}$ on the interval $[-1, 1]$ (see figure). We should be able to find the area of this region using calculus. According to our theory, since $f(x) = \sqrt{1-x^2}$ is continuous, it is integrable so

$$\text{Area} = \int_{-1}^{1} \sqrt{1-x^2}\,dx = \lim_{n\to\infty} \text{Right}(n) = \lim_{n\to\infty} \text{Left}(n).$$

So we should be able to approximate the answer using left and right Riemann sums with increasingly large values of n. Use `xFunctions` to find: Left(5), Right(5), and Midpoint(5). Then Left(52), Right(52), and Midpoint(52). Finally Left(512), Right(512), and Midpoint(512). Correctly round to four decimal places. Note: $\sqrt{1-x^2}$ is typed as `sqrt(1-x^2)`. The image below shows the values for 10 subintervals.



n	Left(n)	Right(n)	Midpoint(n)
5			
52			
512			

c) Are these estimates getting closer to your answer in part (a) as n gets larger?