Math 131 Day 16

My Office Hours: M & W 12:30–2:00, Tu 2:30–4:00, & F 1:15-2:30 or by appointment. **Math Intern** Sun: 12–6pm; M 3-10pm; Tu 2–6, 7–1pm; W and Th: 5-10 pm in Lansing 310. Website: `http://math.hws.edu/~mitchell/Math131S13/index.html`.

Practice

Finish reading 6.4 about the Shell Method to calculate volumes. We will concentrate on rotations around the y-axis, but will illustrate the idea with additional examples about the x-axis. Begin reading Section 6.5 on Length of Curves.

Take home message.

With the disk or slicing method, when the curve is rotated about the y-axis, you need to write the integral in terms of y. This means solving for x in terms of y.

In the same situation, the shell method allows us to use functions of x to determine the volume when rotating around the y axis. We do not have to solve for x. This is an advantage, though sometimes the integrals are harder.

1. Disk Method Practice. Find the volume of the solid that results when the region enclosed by the given curves is revolved about the x-axis.

- **a)** $y = x^2$, $x = 0$, $x = 2$, $y = 0$. (Answer: $32\pi/5$)
- **b)** $y = 1/x$, $x = 1$, $x = 4$, $y = 0$. (Answer: $3\pi/4$)
- **c)** $y = 9 - x^2$, $y = 0$. (Answer: $1296\pi/5$)
- **d)** $y = x^2$, $y = 4x$. (Answer: $2048\pi/15$)
- **e)** $y = \sqrt{x}$, $y = x$. (Answer: $\pi/6$)

2. Shell Method Practice. Page 410–411 #1, 3, 5, 9(shells make this easy), 11.

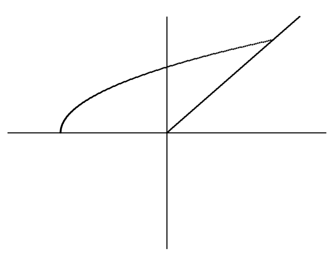
Hand In at Lab

The answers are in this list: $8\pi/3$, $16\pi/3$, $8\pi/15$, $32\pi/15$, $11\pi/6$, $5\pi/6$, 8π. **Be neat and put a box around your answers so I can find them.**

1. **a)** Draw the region R in the first quadrant enclosed by $y = x^2$, $y = 2 - x$ and the x-axis.

- **b)** (Disks) Rotate R about the x-axis and find the resulting volume. [Is it the sum of two pieces or outside minus inside?]
- **c)** (Disks) Rotate R about the y-axis and find the resulting volume. [Is it the sum of two pieces or outside minus inside?]
- **d)** (Shells) Rotate R about the y-axis and and **just set up** the integral for finding the resulting volume.

2. Let R be the region in the upper half-plane bounded by $y = \sqrt{x+2}$, the x axis and the line $y = x$. Find the volume resulting when R is rotated around the x axis. Remember: Outside $-$ Inside. How many pieces do you need?



3. Let S be the region in the first quadrant enclosed by $y = x^2$, $y = 2 - x$ and the y-axis. Note: The region is different than in problem 1.

- **a)** Rotate S about the y-axis. Find the volume using the **shell** method. [Is it the sum of two pieces or a difference?]
- **b)** Rotate S about the y-axis. **Just set up the integral** using the disk method. [Is it the sum of two pieces or outside minus inside?]

OVER

Using the Shell Method.

These are WeBWorK Day 16 problems. But you can show me your answers in lab if you wish.

4. Just set up the integrals for each of the following volume problems. You should try to simplify the integrands where possible.

a) R is the region enclosed by $y = x^2$, $y = x + 2$, and the y axis in the first quadrant. Rotate R about the y-axis.

b) S is the region enclosed by $y = -x^2 + 2x + 3$, $y = 3x - 3$, the y axis, and the x axis in the first quadrant. Rotate S about the y-axis.

c) T is the region enclosed by $y = -x^2 + 2x + 3$, $y = 3x - 3$, and the x axis in the first quadrant. Rotate T about the y-axis.

d) S is the region enclosed by $y = \frac{1}{2}x^2 + 2$ and $y = x^2$ in the first quadrant. Rotate S about the y-axis.

e) T is enclosed by $y = \sqrt{x-2}$, $y = 4 - x$, the y-axis and the x-axis. Rotate T about the y-axis.

f) V is the region enclosed by $y = \sqrt{x-2}$, $y = 4 - x$, and the x-axis. Rotate V about the y-axis.