Math 131 Homework Day 20

My Office Hours: M & W 12:30–2:00, Tu 2:30–4:00, & F 1:15-2:30 or by appointment. Math Intern Sun: 12–6pm; M 3-10pm; Tu 2-6, 7–1pm; W and Th: 5-10 pm in Lansing 310. Website: http://math.hws.edu/~mitchell/Math131S13/index.html.

Practice

For the next few weeks we will be considering a variety of integration techniques. Unless you do lots of practice before and after every class, this material will rapidly become very confusing. Do not fall behind!

I will be giving you several short hand in assignments and WeBWorK assignments to keep you caught up.

Today we continue developing methods of integration. Begin by reviewing the material in Section 4.8 on pages 296–297 which covers basics you already know. (You may ignore formulas 4, 6, and 11 in these lists.) Add to this list the new ones we discussed today: If \( b > 0 \), then

\[
\int b^x \, dx = \frac{1}{\ln b} b^x + c \quad \text{and} \quad \int b^u \, du = \frac{1}{\ln b} b^u + c,
\]

1. The first new technique we will explore is integration by parts which reverses the product rule. Read 7.1 and the online notes. Integration by parts is an important technique the greatly enlarges the number of integrals that you can do. Not all are by parts.
   a) Try page 458 #1, 5, 7, 9, 11[twice!, see Example 3], 13(parts twice), 15, 17, 19, 21. The last few are somewhat harder: try them and come prepared to discuss them.
   b) Try page 458 #37 by shells.

2. Integral Mix Up: Before working these out, go through and classify each by the technique that you think will apply: substitution, parts, parts twice (See Example 3 in the text and class notes), or ordinary methods. Which can’t you do yet? The answers are on the back.
   a) \[ \int 2e^{-\pi x} \, dx \]
   b) \[ \int \cos xe^{\sin x} \, dx \]
   c) \[ \int e^x \cos x \, dx \]
   d) \[ \int x \cos x \, dx \]
   e) \[ \int \cos(2\pi x) \, dx \]
   f) \[ \int \ln x \, dx \]
   g) \[ \int (x^2 + 1)e^{x^3+3x} \, dx \]
   h) \[ \int (x^2 + 1)e^x \, dx \]
   i) \[ \int x^2 \ln x \, dx \]
   j) \[ \int \sec^2(2x) \, dx \]
   k) \[ \int \frac{x}{25 + x^2} \, dx \]
   l) \[ \int \frac{1}{25 + x^2} \, dx \]
   m) \[ \int \frac{1}{\sqrt{1-9x^2}} \, dx \]
   n) \[ \int \frac{\cos x}{\sqrt{1-\sin^2 x}} \, dx \]
   o) \[ \int \arcsin x \, dx \]

Hand In: Spot Check

In some of these problems you will need to use integration by parts twice.

0. WeBWorK set Day20 due late Monday night. Start early.

1. From the list above: #2 (f), (h), and (m)

2. a) Page 458 #12 (Similar to WeBWorK Problem 4.)
   b) Page 458 #22 (This combines two techniques, be patient and careful)
   c) Page 458 #14
   d) Page 458 #26. Remember \( \ln^2 x \) means \( (\ln x)^2 \).

3. a) A tank is formed by rotating the region in the first quadrant enclosed by \( y = (x - 2)^2 \), the \( y \)-axis, and the \( x \)-axis about the \( y \)-axis as shown Set up the integral for the work done pumping oil (density 60 lbs/ft\(^3\)) to the top of the tank.
   b) Set up the work integral if the oil was pumped to a height 2 feet above the top of the tank?
   c) Set up the work integral for pumping the oil to the tank top if there is only 1 foot of oil in the tank.
4. \[ \int \frac{xe^x}{(x + 1)^2} \, dx \] Use \( dv = \frac{1}{(x+1)^2} \, dx \)

2. Some Answers to the Mix-Up Problem: (All \("+c\"\).

<table>
<thead>
<tr>
<th>a) (-\frac{2e^{-\pi x}}{\pi})</th>
<th>b) (e^{\sin x})</th>
<th>c) (\frac{1}{2}e^x(\cos x + \sin x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>d) (x \sin x + \cos x)</td>
<td>e) (\frac{1}{2\pi} \sin(2\pi x))</td>
<td>f) HW</td>
</tr>
<tr>
<td>g) (\frac{1}{3}e^{x^3+3x})</td>
<td>h) HW</td>
<td>i) (\frac{1}{3}x^3 (\ln x - \frac{1}{3}))</td>
</tr>
<tr>
<td>j) (\frac{1}{2} \tan(2x))</td>
<td>k) (\frac{1}{2} \ln(25 + x^2))</td>
<td>l) (\frac{1}{5} \arctan \frac{x}{5})</td>
</tr>
<tr>
<td>m) HW</td>
<td>n) (\arcsin(\sin x))</td>
<td>o) (\frac{1}{2} (\arcsin x)^2)</td>
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