Math 131 Homework Day 21

My Office Hours: M & W 12:30–2:00, Tu 2:30–4:00, & F 1:15-2:30 or by appointment. **Math Intern** Sun: 12–6pm; M 3-10pm; Tu 2–6, 7–1pm; W and Th: 5-10 pm in Lansing 310. Website: `http://math.hws.edu/~mitchell/Math131S13/index.html`.

Practice

We will finish integration by parts today and start on trig integrals. Make sure you do lots of practice. Read Section 7.2. I will not require you to memorize all of these formulas.

1. Today we complete integration by parts is an important technique the greatly enlarges the number of integrals that you can do.

a) Standard parts at least twice: Try page 458ff #23, 25, 27.

b) Definite integrals with parts: Try page 458ff #31, 33, 35.

c) Applications with parts: Try page 458ff #31, 33, 35.

Reference: Summary of Trig Integrals

2. Degree 2 Sine and Cosine Functions. One simple way to do these is to use trig identities.

a) $\displaystyle\int \cos^2 u\, du = \int \tfrac{1}{2} + \tfrac{1}{2}\cos 2u\, du = \tfrac{1}{2}u + \tfrac{1}{4}\sin 2u + c.$

b) $\displaystyle\int \sin^2 u\, du = \int \tfrac{1}{2} - \tfrac{1}{2}\cos 2u\, du = \tfrac{1}{2}u - \tfrac{1}{4}\sin 2u + c.$

3. Low Powers of the Tangent and Secant Functions. These are done with simple identities.

a) $\displaystyle\int \tan u\, du = \ln|\sec u| + c.$

b) $\displaystyle\int \tan^2 u\, du = \int \sec^2 u - 1\, du = \tan u - u + c.$

c) $\displaystyle\int \sec u\, du = \ln|\sec u + \tan u| + c.$

d) $\displaystyle\int \sec^2 u\, du = \tan u + c.$

Reduction Formulas for Large Powers.

These are verified using integration by parts. Repeated application may be necessary.

1) $\displaystyle\int \cos^n u\, du = \tfrac{1}{n}\cos^{n-1} u \sin u + \tfrac{n-1}{n}\int \cos^{n-2} u\, du$

2) $\displaystyle\int \sin^n u\, du = -\tfrac{1}{n}\sin^{n-1} u \cos u + \tfrac{n-1}{n}\int \sin^{n-2} u\, du$

3) $\displaystyle\int \tan^n u\, du = \tfrac{1}{n-1}\tan^{n-1} u - \int \tan^{n-2} u\, du$

4) $\displaystyle\int \sec^n u\, du = \tfrac{1}{n-1}\sec^{n-2} u \tan u + \tfrac{n-2}{n-1}\int \sec^{n-2} u\, du$

Hand In: Spot Check

0. WeBWorK set `Day21` due late Wednesday night. Start early.

1. $\int e^x \cos(7x)\,dx$. Try to avoid fractions until the very end.

2. Page 458 #34. Use a u-substitution first and last. (Messy answer)

3. Page 458 #38. Use shells

4. Page 458 #50. Read the instructions.

5. Page 458 #52. Your choice, either (a) or (b).

6. a) Review: Determine $\int \sin^2(3x)\,dx$.

b) New: Use a reduction formula formula listed above (also see page 463) to determine $\int \cos^4 x\,dx$.

7. Extra Credit: $\int \sin\sqrt{x}\,dx$. Hint: First use a substitution and then use parts.

Math 131 PracTest 2

1. Let S be the region enclosed by the y axis, $y = x^2 + 4$, and $y = 2x^2$ in the first quadrant only.

a) Sketch the region. Find the area of S. (Ans: 16/3)

b) Rotate S about the x-axis and find the resulting volume. (Ans: $512\pi/15$)

c) Rotate S about the y-axis and find the resulting volume. (Ans: 8π)

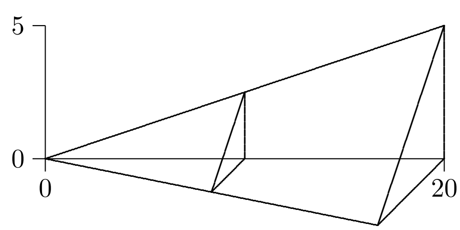
2. Let R be the region in the first quadrant enclosed by enclosed by the x-axis, $y = \sqrt{x}$ and $y = x - 2$. Sketch the region.

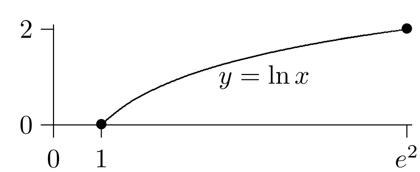
a) Find the area of R. (Ans: 10/3)

b) Rotate R about the x-axis and find the volume. (Ans: $16\pi/3$)

c) Rotate R about the y-axis and find the volume. Try both methods. (Ans: $184\pi/15$)

3. A wooden doorstop with right triangular cross-sections is 20 cm long and 5 cm high at its tall end and 4 cm wide. Find its volume. Hint: Find the equation of the line that forms the top edges and use similar triangles to find the cross-sectional area. (Ans: $\frac{200}{3}$ cm^3.)





4. a) Let R be the region between $y = \ln x$ and the x axis on the interval $[1, e^2]$. Rotate R around the x axis and find the resulting volume. What integration technique should you use? (Answer: $\pi(2e^2 - 1)$.)

b) Rotate R about the y axis and find the volume using the shell method. (Answer: $\frac{\pi}{2}(1 + 3e^4)$.)

c) Rotate R about the y axis and find the volume using the disk method. (Answer: $\frac{\pi}{2}(1 + 3e^4)$.)

5. Let R be the region between $y = \sin \pi x$ and the x axis on the interval $[0, 1]$. Rotate R about the x-axis and find the resulting volume. Hint: Use an identity. (Answer: $\pi/2$.)

6. Find the average value of $f(x) = \sin^3 x$ on the interval $[0, \pi]$. [Ans: $4/3\pi$.]

7. Find the volume of the solid region generated when the area in the first quadrant enclosed by $y = \cos x$, $y = 0$, $x = 0$ and $x = \pi/2$ is revolved around the y-axis. Use shells. [Ans: $\pi^2 - 2\pi$.]

8. **a)** Find $\int_0^{\pi/4} x \tan^2 x\, dx$. [Ans: $\frac{\pi}{4} - \ln\sqrt{2} - \frac{\pi^2}{32}$.]

b) Let R be the region between $y = \tan^2 x$, $x = \pi/4$, and the x-axis in the first quadrant. Rotate R about the y axis and find the volume using the shell method. Re-use part (a). [Ans: $\frac{\pi^2}{2} - 2\pi\ln\sqrt{2} - \frac{\pi^3}{16}$.]

c) Let R be the region enclosed by $y = \tan^{-1} x$, the y-axis, and $y = \pi/4$ in the first quadrant. Rotate R about the y-axis to form a tank. If it is full of a liquid whose density is 64 lbs/cu. ft., how much work is lost if it leaks out the bottom and drops to ground level? Hint: Use part (a)! [Ans: $-16\pi^2 + 64\pi\ln\sqrt{2} + 2\pi^3$.]

9. Find the length of $f(x) = \frac{4}{3}x^{3/2} + 1$ on the interval $[0, 2]$. (Answer: 13/3)

10. Let $f(x) = \frac{1}{6}x^3 + \frac{1}{2}x^{-1}$ on $[2, 3]$. Find the arc length. (Answer: 13/4.)

11. Find the arc length of $f(x) = \ln\cos x$ on $[0, \pi/4]$. (Answer: $\ln|\sqrt{2} + 1|$.)

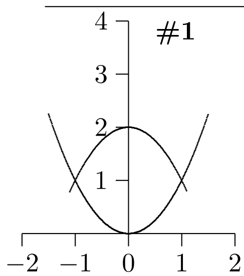
12. Find the volume of the solid region generated when the area in the first quadrant enclosed by $y = \sqrt{\frac{x}{9+x^2}}$, $y = 0$, and $x = 2$ is revolved around the x-axis. [Ans: $\frac{\pi}{2}[\ln\frac{13}{9}]$]

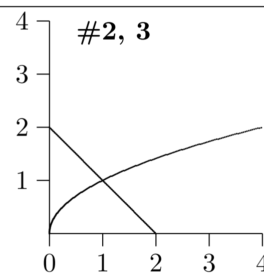
13. Find the volume of the solid region generated when the area in the first quadrant enclosed by $y = \cos x$, $y = 0$, $x = 0$ and $x = \pi/2$ is revolved around the y-axis. Use shells. [Ans: $\pi^2 - 2\pi$.]

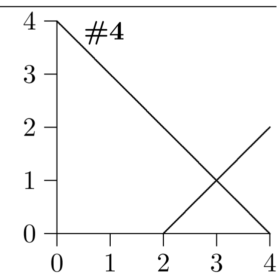
14. **a)** A small farm elevated water tank is in the shape obtained from rotating the region in the first quadrant enclosed by the curves $y = 10 - \frac{1}{2}x^2$, $y = 8$, and the y-axis about the y-axis. Find the work "lost" if the water (62.5 lbs/ft^3) leaks onto the ground from a hole in the bottom of the tank. (Answer: $-6500\pi/3$ ft-lbs.)

b) Find the work "lost" if the water leaks onto the ground from a hole in the side of the tank at height 9 feet. (Answer: $-1750\pi/3$ ft-lbs.)

c) Set up integral for the work to empty a tank containing just one foot of water over the top edge.







1. **Rotation about the x-axis.** Let R be the *entire* region enclosed by $y = x^2$ and $y = 2 - x^2$ in the upper half-plane. Rotate R about the x-axis. The resulting volume is given by:

a) $\pi\int_{-1}^{1}(x^2)^2\,dx - \pi\int_{-1}^{1}(2-x^2)^2\,dx$

b) $\pi\int_{0}^{1}(2-x^2)^2\,dx - \pi\int_{0}^{1}(x^2)^2\,dx$

c) $\pi\int_{-1}^{1}(2-x^2)^2\,dx + \pi\int_{-1}^{1}(x^2)^2\,dx$

d) $\pi\int_{-1}^{1}(2-x^2)^2\,dx - \pi\int_{-1}^{1}(x^2)^2\,dx$

e) $\pi\int_{0}^{1}\left(\sqrt{y}\right)^2\,dy + \pi\int_{1}^{2}\left(\sqrt{2-y}\right)^2\,dy$

f) None of these

2. **Rotation about the x-axis.** Let S be the region enclosed by the x-axis, $y = \sqrt{x}$, and $y = 2 - x$. The volume generated by revolving S about the x-axis is:

a) $\pi\int_{0}^{1}\left(\sqrt{x}\right)^2\,dx - \pi\int_{1}^{2}(2-x)^2\,dx$

b) $2\pi\int_{0}^{1}x(2-x)\,dx - 2\pi\int_{0}^{1}x\sqrt{x}\,dx$

c) $\pi\int_{0}^{1}\left(\sqrt{x}\right)^2\,dx + \pi\int_{1}^{2}(2-x)^2\,dx$

d) $\pi\int_{0}^{1}(2-y)^2 - (y^2)^2\,dy$

e) $\pi\int_{0}^{2}\left(\sqrt{x}\right)^2\,dx - \pi\int_{1}^{2}(2-x)^2\,dx$

f) None of these

3. Rotation about the y-axis. Let T be the region enclosed by the y-axis, $y = \sqrt{x}$, and $y = 2 - x$ (a different region than in Problem 2). The volume generated by revolving T about the y-axis is:

a) $\pi \int_0^2 (y^2)^2\, dy - \pi \int_0^2 (2-y)^2\, dy$

b) $2\pi \int_0^1 x(2-x)^2\, dx - 2\pi \int_0^1 x\left(\sqrt{x}\right)^2\, dx$

c) $\pi \int_0^1 \left(\sqrt{x}\right)^2\, dx + \pi \int_1^2 (2-x)^2\, dx$

d) $\pi \int_0^1 (2-y)^2\, dy + \pi \int_1^2 (y^2)^2\, dy$

e) $2\pi \int_0^1 x(2-x)\, dx - 2\pi \int_0^1 x(\sqrt{x})\, dx$

f) None of these

4. Rotation about the y-axis. Let U be the region enclosed by the y-axis, the x-axis, $y = x - 2$, and $y = 4 - x$. The volume generated by revolving U about the y-axis is:

a) $\pi \int_0^1 (y-2)^2 + \pi \int_1^4 (4-y)^2\, dy$

b) $2\pi \int_0^3 x(4-x)^2\, dx - 2\pi \int_2^3 x(x-2)^2\, dx$

c) $2\pi \int_0^4 x(4-x)\, dx - 2\pi \int_2^3 x(x-2)\, dx$

d) a and c

e) a and b

f) None of these

Practest 2, Part 2.

1. Integral Mix Up: Gotta game plan?

a) $\int (4x^3+1)\ln x\, dx$ **b)** $\int xe^{x+1}\, dx$ **c)** $\int e^{2x}\cos x\, dx$

d) $\int x\sec^2 x\, dx$ **e)** $\int \cos^3(4x)\, dx$

f) $\int \ln(2x^3)\, dx$ **g)** $\int \sin^2(5x)\, dx$ **h)** $\int x^2 \ln x^2\, dx$

i) $\int \arctan 2x\, dx$ **j)** $\int 2\sec^3 x \tan x\, dx$ See hint **k)** $\int x\sin^2 x\, dx$ **l)** $\int \tan^3 x\, dx$

Hint: $2\sec^3 x \tan x = 2\sec^2 x \sec x \tan x$

2. Find the area enclosed by the curves $y = x^3 + x$ and $y = 3x^2 - x$.

3. Determine $\int 2x \arcsin x\, dx$. You will need to use several different methods.

4. Integral Mix Up: Before working these out, go through and classify each by the technique that you think will apply: substitution, parts, parts twice, trig methods, or ordinary methods.

a) $\int xe^{2x}\, dx$ **b)** $\int xe^{x^2+1}\, dx$ **c)** $\int e^x \cos x\, dx$

d) $\int x^2\sin x\, dx$ **e)** $\int \sin^4(2x)\, dx$ **f)** $\int x\cos(x^2)\, dx$ **g)** $\int \frac{(\ln x)^2}{x}\, dx$

h) $\int \ln(x^2)\, dx$ **i)** $\int \ln\sqrt{x}\, dx$ **j)** $\int x^2 \ln x\, dx$ **k)** $\int \cos^2(12x)\, dx$

l) $\int \tan x\, dx$ **m)** $\int \arctan x\, dx$ **n)** $\int \sin(5x)\cos(2x)\, dx$

o) $\int x\sqrt{4-x^2}\, dx$ **p)** $\int \frac{1}{\sqrt{9-4x^2}}\, dx$