Math 131 Homework Day 22

My Office Hours: M & W 12:30–2:00, Tu 2:30–4:00, & F 1:15-2:30 or by appointment. **Math Intern** Sun: 12–6pm; M 3-10pm; Tu 2–6, 7–1pm; W and Th: 5-10 pm in Lansing 310. Website: `http://math.hws.edu/~mitchell/Math131S13/index.html`.

Practice

Re-read Section 7.2 on trig integrals. Review the material on this sheet. Do **lots of practice** before and after class, or else this material will rapidly become very confusing. *Read today's online notes for more examples.* **Begin Section 7.3 on Triangle Substitution and read the handout over break**

1. Try page 466 # 1, 3, 5, 9–21 (odd), 25, and 29. Ask about them in lab if you get stuck.

Reference: Summary of Trig Integrals

1. Degree 2 Sine and Cosine Functions. One simple way to do these is to use trig identities.

a) $\int \cos^2 u\,du = \int \frac{1}{2} + \frac{1}{2}\cos 2u\,du = \frac{1}{2}u + \frac{1}{4}\sin 2u + c.$

b) $\int \sin^2 u\,du = \int \frac{1}{2} - \frac{1}{2}\cos 2u\,du = \frac{1}{2}u - \frac{1}{4}\sin 2u + c.$

2. Low Powers of the Tangent and Secant Functions. These are done with simple identities.

a) $\int \tan u\,du = \int \frac{\sin u}{\cos u}\,du = \ln|\sec u| + c.$

b) $\int \tan^2 u\,du = \int \sec^2 u - 1\,du = \tan u - u + c.$

c) $\int \sec u\,du = \int \frac{\sec^2 u + \sec u \tan u}{\sec u + \tan u}\,du = \ln|\sec u + \tan u| + c.$

d) $\int \sec^2 u\,du = \tan u + c.$

Hand In at Lab

I will **collect these** not just check them off.

1. a) Fill in the table on the left of values for the sine and cosine functions. Use exact values, not decimal approximations.

b) Fill in the table on the right using the half-angle from the front of your text.

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$					
$\cos\theta$					

$\cos^2(3\theta)$	
$\sin^2(4\theta)$	
$\cos^2(-2\theta)$	

2. Determine the following antiderivatives. All of these are WeBWorK Day 22 Pre-lab problems, so check your answers.

a) $\int \sin^2(4t)\,dt$ via a half-angle trig identity formula .

b) $\int \cos^5(x)\,dx$ via a reduction formula.

c) $\int \sin^2(4x)\cos^3(4x)\,dx$ via a trig identity. Remember to convert to u. (See back.)

d) $\int \sin^3(2x)[\cos(2x)]^{-4}\,dx$ via a trig identity. Remember to convert to u. (See back.)

Reduction Formulas for Large Powers.

These are verified using integration by parts. Repeated application may be necessary.

1) $\displaystyle\int \cos^n u\, du = \frac{1}{n}\cos^{n-1} u \sin u + \frac{n-1}{n}\int \cos^{n-2} u\, du$

2) $\displaystyle\int \sin^n u\, du = -\frac{1}{n}\sin^{n-1} u \cos u + \frac{n-1}{n}\int \sin^{n-2} u\, du$

3) $\displaystyle\int \tan^n u\, du = \frac{1}{n-1}\tan^{n-1} u - \int \tan^{n-2} u\, du$

4) $\displaystyle\int \sec^n u\, du = \frac{1}{n-1}\sec^{n-2} u \tan u + \frac{n-2}{n-1}\int \sec^{n-2} u\, du$

Guidelines for Products of Sines and Cosines:

These general principles can help you solve integrals of the form $\int \sin^m x \cos^n x\, dx$.

1. If the power of sine is odd and positive, split off a factor of sine for du and convert the rest to cosines, let $u = \cos x$, and then integrate. For example,

$$\int \overbrace{\sin^{2k+1} x}^{m=2k+1 \text{ odd}} \cos^n x\, dx = \int (\sin^2 x)^k x \cos^n x \cdot \overbrace{\sin x\, dx}^{\text{use for } du} = \int \overbrace{(1-\cos^2 x)^k}^{\text{convert to cosines}} \cos^n x \cdot \sin x\, dx = -\int (1-u^2)^k u^n\, du.$$

2. If the power of cosine is odd and positive (and the power of sine is even), split off a factor of cosine for du and convert the rest to sines, let $u = \sin x$, and then integrate. For example,

$$\int \sin^m x \overbrace{\cos^{2k+1} x}^{n=2k+1 \text{ odd}}\, dx = \int \sin^m x (\cos^2 x)^k \cdot \overbrace{\cos x\, dx}^{\text{use for } du} = \int \sin^m x \overbrace{(1-\sin^2 x)^k}^{\text{convert to sines}} \cdot \cos x\, dx = \int u^m (1-u^2)^k\, du.$$

3. If both powers of sine and cosine are *even* and non-negative, make repeated use of the identities $\sin^2 u = \frac{1}{2} - \frac{1}{2}\cos 2u$ and $\cos^2 u = \frac{1}{2} + \frac{1}{2}\cos 2u$ to powers of cosines. Then use reduction formula #1.

4. Use a table of integrals or *WolframAlpha* or other software. Certainly you should use this tool in later courses whether in math or other departments.

Guidelines for Products of Tangents and Secants:

These general principles can help you solve integrals of the form $\int \tan^m x \sec^n x\, dx$.

1. If the power of secant is *even* and positive, split off $\sec^2 x$ to use for du and convert the rest to tangents, then let $u = \tan x$, and integrate. For example,

$$\int \tan^m x \overbrace{\sec^{2k} x}^{n=2k \text{ even}}\, dx = \int \tan^m x (\sec^2 x)^{k-1} \cdot \overbrace{\sec^2 x\, dx}^{\text{use for } du} = \int \tan^m x \overbrace{(1+\tan^2 x)^{k-1}}^{\text{convert to tangents}} \cdot \sec^2 x\, dx = \int u^m (1+u^2)^{k-1}\, du.$$

2. If the power of tangent is odd and positive (and the power of secant is odd), split off $\sec x \tan x$ for du and convert the rest to secants, let $u = \sec x$, and then integrate. For example,

$$\int \overbrace{\tan^{2k+1} x}^{m=2k+1 \text{ odd}} \sec^n x\, dx = \int (\tan^2 x)^k \sec^{n-1} x \cdot \overbrace{\sec x \tan x\, dx}^{\text{use for } du} = \int \overbrace{(\sec^2 x - 1)^k}^{\text{convert to secants}} \sec^{n-1} x \cdot \sec x \tan x\, dx = \int (u^2-1)^k u^{n-1}\, du.$$

3. If m is even and n is odd, convert the tangents to secants and use the reduction formula above:

$$\int \overbrace{\tan^{2k} x}^{m=2k \text{ even}} \overbrace{\sec^n x}^{n \text{ odd}}\, dx = \int \overbrace{(\sec^2 x - 1)^k}^{\text{convert to secants}} \sec^n x\, dx.$$

4. In real life, use *WolframAlpha*, or look in a table of integrals.