Math 131 Day 30

My Office Hours: M & W 12:30–2:00, Tu 2:30–4:00, & F 1:15-2:30 or by appointment. **Math Intern** Sun: 12–6pm; M 3-10pm; Tu 2–6, 7–10pm; W and Th: 5-10 pm in Lansing 310. Website: `http://math.hws.edu/~mitchell/Math131S13/index.html`.

Practice

Read 8.1 and 8.2. The rest of the term will be spent working in Chapters 8 and 9 on sequences and series. This is fun stuff! Some very interesting ideas. But much of the terminology will be new to you so you really need to stay on top of this material.

1. These practice problems familiarize you with sequence terminology.

a) Try page 546 Review Questions #1 through 5. These are very good ones!

b) These have you calculating limits of sequences. Page 546 # 9, 11, 13, 15, 19, 21, 25, 23, 35, 37, 39, 40, 41.

2. Review: Determine these three integrals; for one use a theorem to make it quick. (Potential Answers: Diverge, 1, 2, 1/3, 1/4).

a) $\displaystyle\int_0^\infty \frac{x}{(1+x^2)^{3/2}}\,dx$ **b)** $\displaystyle\int_1^\infty \frac{1}{x^4}\,dx$ **c)** $\displaystyle\int_0^1 \frac{2}{2x-x^2}$

3. Evaluate the following using highest powers and absolute values: $\displaystyle\lim_{x\to-\infty} \frac{\sqrt{16x^2-x+6}}{4-2x}$

Summary of Key Limits

You should know and be able to use all of the following limits.

1. $\displaystyle\lim_{n\to\infty} \left(1+\frac{k}{n}\right)^n = e^k$. In particular $\displaystyle\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e$.

2. $\displaystyle\lim_{n\to\infty} n^{1/n} = \lim_{n\to\infty} \sqrt[n]{n} = 1$.

3. $\displaystyle\lim_{n\to\infty} \frac{n!}{n^n} = 0$ and $\displaystyle\lim_{n\to\infty} \frac{n^n}{n!} = \infty$ (diverges).

4. Consider the sequence $\{r^n\}_{n=1}^\infty$, where r is a real number.

a) If $|r| < 1$, then $\displaystyle\lim_{n\to\infty} r^n = 0$;

b) If $r = 1$, then $\displaystyle\lim_{n\to\infty} r^n = 1$;

c) Otherwise ($|r| > 1$ or $r = -1$), we have $\displaystyle\lim_{n\to\infty} r^n$ does not exist (diverges).

Hand In

Work on WeBWorK `Day 30` due Thursday. WeBWorK `Day 29` is due Tuesday.

1. Simplify each of these expressions that involve factorials.

a) $\dfrac{10!}{8!}$ **b)** $\dfrac{5!}{7!}$ **c)** $\dfrac{(n+2)!}{n!}$ **d)** $\dfrac{3^2\cdot(n-1)!}{(n+1)!}$ **e)** $\dfrac{4^4}{4!}$

2. Evaluate the limits of the following sequences or show that they diverge. Use **key limits** where appropriate. Use previous methods and L'Hopital's rule as needed (after converting to x.)

a) $\displaystyle\left\{\left(1+\frac{2}{n}\right)^n\right\}_{n=1}^\infty$ **b)** $\displaystyle\left\{\left(\frac{1}{n}\right)^{1/n}\right\}_{n=1}^\infty$ **c)** $\left\{n\sin\left(\frac{1}{n}\right)\right\}_{n=1}^\infty$ **d)** $\left\{\left(\frac{e}{\pi}\right)^n\right\}_{n=1}^\infty$

3. Now find the limit of $\left\{\ln(n)+\ln\left(\sin\left(\frac{1}{n}\right)\right)\right\}_{n=1}^\infty$. HINT: Use a log property to simplify the expression and then use earlier work.

4. Close Reading. Very carefully read page 539 in the text where the terms **non-increasing**, **non-dcreasing**, **monotonic**, and **bounded** are defined. Make up your own (easy) examples of the following:

a) Give an example of a non-increasing sequence with a limit. (Evaluate the limit.) Is your sequence bounded? If so, give a bound B.

b) Give an example of a non-decreasing sequence withOUT a limit. (Show it diverges.) Is your sequence bounded? If so, give a bound B.

5. Now look at the BACK of the orange sheet of EXAMPLES OF SEQUENCES. Using the graphs list all of the adjectives that appear to apply to each: **non-increasing**, **non-decreasing**, **monotonic**, and **bounded**. If the sequence is bounded, give a bound B which appears to work.

a) Upper left: $\left\{(1+n^2)^{1/n}\right\}_{n=1}^{\infty}$

b) Upper right: $\left\{\sum_{k=1}^{n}(-1)^{n+1}\frac{1}{n}\right\}_{n=1}^{\infty}$

c) Lower left: $\left\{\frac{4^n}{n!}\right\}_{n=1}^{\infty}$

d) Lower right: $\left\{\frac{1+(-1)^n}{n}\right\}_{n=1}^{\infty}$

e) We say that a sequence $\{a_n\}$ is **eventually monotonic** if there is some index N so that for all $n \geq N$, $\{a_n\}_{n=N}^{\infty}$ is monotonic. In other words, from the Nth term on the sequence is either non-decreasing or non-increasing. Which of the four sequences above are eventually monotonic? Give the N for those that are. (Monotonic sequences are automatically eventually monotonic.)

f) List two sequences from the FRONT of the orange sheet that are NOT eventually monotonic.

6. Review: Evaluate $\int_1^2 \frac{1}{\sqrt{x^2-1}}\,dx$. If it is improper, use correct notation. What integration technique is required?

7. Extra Credit. Find these limit (if they exists):

a) $\{a_n\}_{n=1}^{\infty} = \left\{\frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}+\cdots+\frac{n}{n^2}\right\}_{n=1}^{\infty}$

b) $\{a_n\}_{n=2}^{\infty} = \left\{\int_1^{\infty}\frac{1}{x^n}\,dx\right\}_{n=2}^{\infty}$