Math 131 Day 35

My Office Hours: M & W 12:30–2:00, Tu 2:30–4:00, & F 1:15-2:30 or by appointment. **Math Intern** Sun: 12–6pm; M 3-10pm; Tu 2–6, 7–10pm; W and Th: 5-10 pm in Lansing 310. Website: `http://math.hws.edu/~mitchell/Math131S13/index.html`.

Reading and Practice

Review all of Section 8.5 on the ratio, root, and comparison tests. Begin reading Section 8.6 on Alternating Series.

Hand In

Justify your answers with an argument. Be sure you explain why the series you are comparing to converges or diverges.

1. Review: Divergence Test: Page 567 #18.

2. Review: Integral Test: Page 567 #28.

3. The next few are comparisons (direct or limit):

a) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^{3/2}+1}$.

b) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{3k-\sqrt{k}}$.

c) Page 576 #68(a). Consider $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k}$.

4. Finish with the Ratio Test:

a) Page 576 #10

b) Page 576 #12

c) and $\displaystyle\sum_{n=1}^{\infty} \frac{k!}{9^k}$

5. Your Choice: Page 576 #42.

6. Redo Page 567 #18 with a test that will answer the question.

7. WeBWorK Day 35 Due Tuesday.

Eight Tests

1. Ratio Test. Assume that $\displaystyle\sum_{n=1}^{\infty} a_n$ is a series with **positive** terms and let $r = \displaystyle\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$.

1. If $r < 1$, then the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.
2. If $r > 1$ or $\displaystyle\lim_{n\to\infty} \frac{a_{n+1}}{a_n} > \infty$, then the series $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges.
3. If $r = 1$, then the test is inconclusive. The series may converge or diverge.

2. Root Test. Assume that $\displaystyle\sum_{n=1}^{\infty} a_n$ is a series with **positive** terms and let $r = \displaystyle\lim_{n\to\infty} \sqrt[n]{a_n}$.

1. If $r < 1$, then the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.
2. If $r > 1$ or $\displaystyle\lim_{n\to\infty} \frac{a_{n+1}}{a_n} > \infty$, then the series $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges.
3. If $r = 1$, then the test is inconclusive. The series may converge or diverge.

3. Limit Comparison Test. Assume that $a_n > 0$ and $b_n > 0$ for all n (or at least all $n \geq k$) and that $\lim_{n\to\infty} \frac{a_n}{b_n} = L$.

1) If $0 < L < \infty$ (i.e., L is a positive, *finite* number), then either $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ *both* converge or *both* diverge.

2) If $L = 0$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

3) If $L = \infty$ and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges.

4. Direct Comparison Test. Assume $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series with positive terms.

a) If $0 < a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges. (If the bigger series converges, so does the smaller series.)

b) If $0 < b_n \leq a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges. (If the smaller series diverges, so does the bigger series.)

5. The Geometric Series Test.

a) If $|r| < 1$, then the geometric series $\sum_{n=0}^{\infty} ar^n$ converges to $\frac{a}{1-r}$.

b) If $|r| \geq 1$, then the geometric series $\sum_{n=0}^{\infty} ar^n$ diverges.

6. The nth term test for Divergence. If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges. (If $\lim_{n\to\infty} a_n = 0$, this test is useless.)

7. The Integral Test. If $f(x)$ is a **positive**, **continuous**, and **decreasing** for $x \geq 1$ and $f(n) = a_n$, then $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)\,dx$ either both converge or both diverge.

8. The p-series Test. The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1. \end{cases}$

Class Exercise: Rate These Arguments

Each of the following statements is an attempt to show that a given series is convergent or divergent using the Comparison Test. Classify each statement, 'correct' if the argument is valid, or 'incorrect' if any part of the argument is flawed. (Note: Even if the conclusion is true but the argument that led to it was wrong, classify it as incorrect.)

a) For all $n \geq 3$ we have $\ln n > 1$, so $0 \leq \frac{1}{n} < \frac{1}{n\ln(n)}$, and the series $\sum_{n=3}^{\infty} \frac{1}{n}$ diverges by the p-series test ($p = 1$), so by the Comparison Test, the series $\sum_{n=3}^{\infty} \frac{1}{n\ln(n)}$ diverges.

b) For all $n \geq 1$ we have $\sqrt{n+1} > 1$, so $0 < \frac{1}{n} < \frac{\sqrt{n+1}}{n}$ and the series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by the p-series test ($p = 1$), so by the Comparison Test, the series $\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{n}$ diverges.

c) For all $n > 2$, $0 < \frac{n}{3-n^3} < \frac{1}{n^2}$, and the series $\sum_{n=2}^{\infty} \frac{1}{n^2}$ converges by the p-series test ($p = 2 > 1$), so by the Comparison Test, the series $\sum_{n=2}^{\infty} \frac{n}{3-n^3}$ converges.

d) For all $n \geq 1$, $0 < \frac{\cos^2(n)}{n^3} < \frac{1}{n^3}$, and the series $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges by the p-series tes ($p = 3 > 1$)t, so by the Comparison Test, the series $\sum_{n=1}^{\infty} \frac{\cos^2(n)}{n^3}$ converges.

e) For all $n \geq 1$, $0 < \frac{1}{n^2} < \frac{2n+1}{n^3}$, and the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by the p-series test ($p = 2 > 1$), so by the Comparison Test, the series $\sum_{n=1}^{\infty} \frac{2n+1}{n^3}$ converges.