Math 131 Day 36

No new WeBWorK. My Office Hours: M & W 12:30–2:00, Tu 2:30–4:00, & F 1:15-2:30 or by appointment. **Math Intern** Sun: 12–6pm; M 3-10pm; Tu 2–6, 7–10pm; W and Th: 5-10 pm in Lansing 310. Website: `http://math.hws.edu/~mitchell/Math131S13/index.html`.

Practice

1. a) **Review** all of Section 8.5. The Root test is new today.

b) Begin to read Section 8.6 on Alternating Series 577–580. Skip the subsection on remainders. But do read pages 582–583 on Abslolute Convergence.

2. Try page 576 #19, 21, 23.

3. Try page 585 #11, 13, 15, 17, 23.

The Newest Tests

1. The Ratio Test. Assume that $\sum_{n=1}^{\infty} a_n$ is a series with **positive** terms and let $r = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$.

1. If $r < 1$, then the series $\sum_{n=1}^{\infty} a_n$ converges.
2. If $r > 1$ or $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} > \infty$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.
3. If $r = 1$, then the test is inconclusive. The series may converge or diverge.

2. The Root Test. Assume that $\sum_{n=1}^{\infty} a_n$ is a series with **positive** terms and let $r = \lim_{n\to\infty} \sqrt[n]{a_n}$.

1. If $r < 1$, then the series $\sum_{n=1}^{\infty} a_n$ converges.
2. If $r > 1$ or $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} > \infty$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.
3. If $r = 1$, then the test is inconclusive. The series may converge or diverge.

3. The Alternating Series Test. Assume $a_n > 0$. The alternating series $\sum_{n=1}^{\infty}(-1)^n a_n$ converges if the following two conditions hold:

a) $\lim_{n\to\infty} a_n = 0$

b) $a_{n+1} \leq a_n$ for all n (i.e., a_n is decreasing). [Note: This can be checked by taking the derivative.]

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Hand In—Be Especially Neat and Careful

1. Determine whether the following arguments are correct. If not, indicate where there is an error.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n^2+n}$. ARGUMENT: Direct comparison test with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$. Check: For $n \geq 1$ we have $0 < \frac{2}{n^2+n} < \frac{1}{n}$. Since $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by the p-series test ($p = 1$), then $\sum_{n=1}^{\infty} \frac{2}{n^2+n}$ diverges by the direct comparison test.

b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\arctan n}$. ARGUMENT: Alternating Series test with $a_n = \frac{1}{\arctan n}$. Check the two conditions.

1. $\displaystyle\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{\arctan n} = 0$ checks.
2. Decreasing? Here's one way to check: Take the derivative. Let $f(x) = \frac{1}{\arctan x} = (\arctan x)^{-1}$. Then using the chain rule $f'(x) = -1(\arctan x)^{-2} \cdot \frac{1}{1+x^2} < 0$. So the function and sequence are decreasing.

   Since the series satisfies the two hypotheses, by the Alternating Series test, $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$ converges.

2. Page 576 #18. Make sure to check the necessary conditions to apply the test.

3. a) Page 576 #20. Make sure to check the necessary conditions to apply the test.

b) Page 576 #22. Be careful of the algebra. What is $(a^{k^2})^{1/k}$? Make sure to check the necessary conditions to apply the test.

c) Page 576 #24. Make sure to check the necessary conditions to apply the test.

4. a) Choose your test: Does $\displaystyle\sum_{n=1}^{\infty} \frac{(k!)^2}{(2k)!}$ converge or diverge? Make sure to check the necessary conditions to apply the test.

b) Page 576 #48. Make sure to check the necessary conditions to apply the test.

c) Choose your test: Does $\displaystyle\sum_{k=1}^{\infty} \frac{2^k k!}{k^k}$ converge or diverge? This a a great one! Make sure to check the necessary conditions to apply the test.

5. a) Page 585 #12 (What two conditions must you check?)

b) Page 585 #14 (What two conditions must you check?)

c) Does $\displaystyle\sum_{k=1}^{\infty} \cos(k\pi)\frac{k^2}{2k^2+3}$ converge or diverge? (What is $\cos(\pi k)$ is another way to write what?)

Practice: Many of these will be on Lab

First determine which test you will apply to each. What conditions do you need to check? Finally, justify your answers with an ARGUMENT: Many of these can analyzed using ratio, root, limit and direct comparison tests. A few may require other tests such as: nth term test, p-series test, integral test, or geometric series test. [Try to avoid the integral test since that involves a lot of work.]

a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2+4n}}$ **b)** $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan n}{1+n^2}$ **c)** $\displaystyle\sum_{n=1}^{\infty} \frac{3n}{(n+1)2^n}$ **d)** $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3n+1}{n+2}\right)^n$

e) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{n^3}$ **f)** $\displaystyle\sum_{n=1}^{\infty} \frac{5\cdot 6^n}{4^{2n}}$ **g)** $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n^2-5n+4}$ **h)** $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{2^n}$

i) $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{n^2+1}$ **j)** $\displaystyle\sum_{n=1}^{\infty} \frac{5n^7-1}{2n^9+1}$ **k)** $\displaystyle\sum_{n=1}^{\infty} \left(1+\frac{4}{n}\right)^n$ **l)** $\displaystyle\sum_{n=1}^{\infty} \left(1+\frac{4}{n}\right)^{n^2}$

m) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$ **n)** $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{3^n+2}$ **o)** $\displaystyle\sum_{n=1}^{\infty} \ln(2n+3)-\ln(3n+2)$ **p)** $\displaystyle\sum_{n=1}^{\infty} \frac{5n^7-2n}{2n^8+11n^2}$ **q)** $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{8/7}}$