Math 131 Day 39

My Office Hours: M & W 12:30–2:00, Tu 2:30–4:00, & F 1:15-2:30 or by appointment. **Math Intern** Sun: 12–6pm; M 3-10pm; Tu 2–6, 7–10pm; W and Th: 5-10 pm in Lansing 310. Website: `http://math.hws.edu/~mitchell/Math131S13/index.html`.

Practice

Read Section 9.2 on **power series**. **Skim** the first few pages (to page 594) of Section 9.1 on Taylor polynomials.

1. Vocabulary: power series, radius of convergence, interval of convergence.

2. Practice with radius and interval of convergence: Try page 609 #3, 7, 9, 11, 13, 15 and 17.

Key Results

1. The Ratio Test Extension. Assume that $\sum_{n=1}^{\infty} a_n$ is a series with **non-zero** terms and let $r = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$.

1. If $r < 1$, then the series $\sum a_n$ converges *absolutely*.
2. If $r > 1$ (including ∞), then the series $\sum a_n$ diverges.
3. If $r = 1$, then the test is inconclusive. The series may converge or diverge.

2. Convergence of Power Series. For a power series $\sum_{n=0}^{\infty} a_n(x-c)^n$ centered at c, precisely one of the following is true.

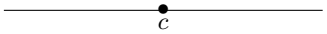
a) The series converges only at $x = c$. $(R = 0)$

b) There is a real number $R > 0$ so that the series converges absolutely for $|x - c| < R$ and diverges for $|x - c| > R$.

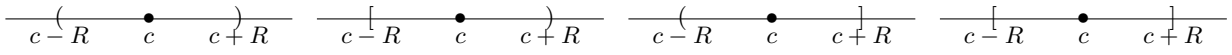
c) The series converges for all x. $(R = \infty)$

NOTE: In case (b) the power series may converge at both endpoints, either endpoint, or neither endpoint. You have to check the convergence at the endpoints separately. Here's what the intervals of convergence can look like:

a) $R = 0$:

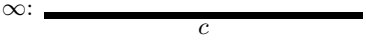


b) $R \neq 0, \infty$:



$(c-R, c+R)$ $\quad$ $[c-R, c+R)$ $\quad$ $(c-R, c+R]$ $\quad$ $[c-R, c+R]$

c) $R = \infty$:



Hand In

0. WeBWorK: `Day 38` due Tuesday. `Day 39` due Thursday.

1. Find the fourth Taylor polynomial $p_4(x)$ for $\ln(1-x)$ centered at $a = 0$. Bonus: What would $p_\infty(x)$ be? Write your answer as a series.

2. Does $\sum \frac{(n+2)!}{(-9)^{n+1}}$ converge absolutely, conditionally, or diverge. Use the extension.

3. a) Determine the **Radius of Convergence**: Page 609 #10

b) Assuming we get this far: Determine the **Interval of Convergence** for the same series.

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4. a) Determine the **Radius** for $\sum_{n=1}^{\infty} 5n(x-4)^n$.

b) Assuming we get this far: Determine the **Interval of Convergence** for the same series

5. a) Determine the **Radius of Convergence** for $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{2n}$.

b) Assuming we get this far: Determine the **Interval of Convergence** for the same series

6. a) Determine the **Radius of Convergence** for $\sum_{n=1}^{\infty} \frac{3^n x^n}{n!}$. Be careful to interpret your answer correctly.

b) Assuming we get this far: Determine the **Interval of Convergence** for the same series