Math 131 Day 40

My Office Hours: M & W 12:30–2:00, Tu 2:30–4:00, & F 1:15-2:30 or by appointment. **Math Intern** Sun: 12–6pm; M 3-10pm; Tu 2–6, 7–10pm; W and Th: 5-10 pm in Lansing 310. Website: `http://math.hws.edu/~mitchell/Math131S13/index.html`.

Practice

Read Section 9.2 on **power series**. Read the online notes. Read pages 611–614 on Taylor Series.

1. Vocabulary: power series, radius of convergence, interval of convergence.

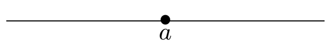
2. Practice with radius and interval of convergence: Try page 609 #3, 7, 9, 11, 13, 15 and 17.

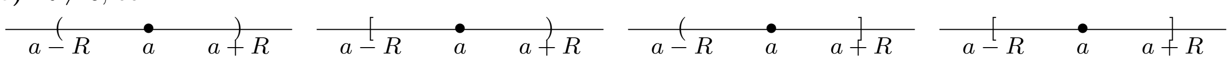
Key Results

1. Convergence of Power Series. For a power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ centered at a, precisely one of the following is true.

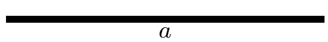
a) The series converges only at $x = a$. ($R = 0$)
b) There is a real number $R > 0$ so that the series converges absolutely for $|x-a| < R$ and diverges for $|x-a| > R$.
c) The series converges for all x. ($R = \infty$)

NOTE: In case (b) the power series may converge at both endpoints, either endpoint, or neither endpoint. You have to check the convergence at the endpoints separately. Here's what the intervals of convergence can look like:

a) $R = 0$:


b) $R \neq 0, \infty$:


$(a-R, a+R)$ $[a-R, a+R)$ $(a-R, a+R]$ $[a-R, a+R]$

c) $R = \infty$:


Hand In

Be neat. Carefully justify your work. Make this your best assignment.

0. WeBWorK: `Day 40` due Sunday. `Day39B` due Thursday.

1. a) Find the fourth Taylor polynomial $p_4(x)$ for $f(x) = \frac{1}{1-x} = (1-x)^{-1}$ centered at $a = 0$.
b) What would $p_\infty(x)$ be? Write your answer as a series. Bonus: Find its radius of convergence.

2. Finding the Radius and Interval of Convergence. This requires finding the radius of convergence and then checking the endpoints. **Note:** There are lots of similar problems on line in the notes.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(x-2)^n}{n2^n}$.

b) $\displaystyle\sum_{n=0}^{\infty} \frac{3^n x^{n+1}}{(2n)!}$.

c) $\displaystyle\sum_{k=0}^{\infty} k!(x+4)^k$.

d) EZ Bonus: $\displaystyle\sum_{k=1}^{\infty} \frac{k(x-10)^k}{3^k}$.

3. XC for Xtra Practice: Find the third Taylor polynomial $p_3(x)$ for $f(x) = \sqrt{x} = x^{1/2}$ centered at $a = 1$.

Review Exercises

4. Determine whether $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[5]{n^4+3}}$ converges conditionally, absolutely, or not at all. What strategy may save you work?

5. Determine whether $\sum_{n=1}^{\infty} \frac{(-1)^n 3^n}{(2n)!}$ converges conditionally, absolutely, or not at all. What strategy may save you work?

6. Find the fourth Taylor polynomial for $p_4(x)$ for $\cos x$ centered at $a = 0$.

7. Determine the radius and interval of convergence for the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (x-3)^n}{n}$.

8. Determine the radius and interval of convergence for the power series $\displaystyle\sum_{n=0}^{\infty} \frac{4^n x^{2n}}{n^2}$.

9. Determine the radius and interval of convergence for the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{(2n)!}$.