Math 131 PracTest 1

0. Be sure to review Lab 4 (and all labs). There are lots of good questions on it.

1. a) State the Mean Value Theorem and draw a graph that illustrates.

b) Name an important theorem where the Mean Value Theorem was used in the proof.

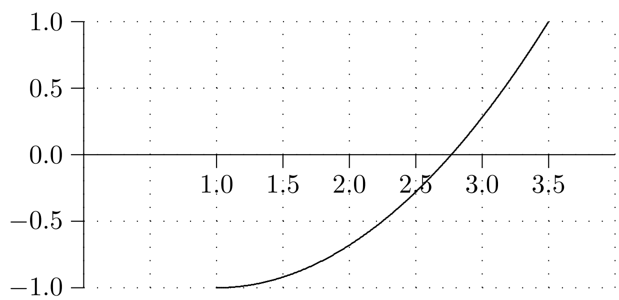
c) In your own works, state the definition of the definite integral of a continuous function on a closed interval $[a, b]$ as a limit of some process.

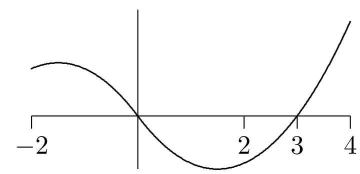
d) State the Fundamental Theorem of Calculus (FTC I).

e) Determine $\frac{d}{dx}\left[\int_{x^3}^{1} \sin^2(t)\,dt\right]$. What theorem(s) apply?

f) If $\int_0^x g(t)\,dt = \frac{\sin x}{e^x}$, what is $g(0)$? Explain.

2. Draw and then estimate Right(5) for the graph of f on $[1, 3.5]$ on the left below. Be careful of the scale.





3. In the graph of f on the right above, assume that $\int_{-2}^{4} f(x)\,dx = 1$, $\int_{-2}^{0} f(x)\,dx = 3$, and $\int_{-2}^{3} f(x)\,dx = -1.2$. Evaluate the following.

a) $\int_3^4 f(x)\,dx$ **b)** $\int_0^3 f(x)\,dx$ **c)** $\int_{-1}^{5} 2f(x-1)\,dx$ **d)** $\int_{-2}^{0} f(x) - 4\,dx$

e) If $f(x)$ is symmetric about the origin (an 'odd' function), what is $\int_0^2 f(x)\,dx$?

4. Calculate these "look-alike" indefinite integrals.

a) $\int \sqrt{4t-1}\,dt$ **b)** $\int \frac{1}{\sqrt{25-4t^2}}\,dt$ **c)** $\int \frac{t}{\sqrt{1-4t^2}}\,dt$ **d)** $\int t\sqrt{4-t}\,dt$ **e)** $\int \frac{t}{1+4t^4}\,dt$

f) $\int \frac{t^3}{1+4t^4}\,dt$ **g)** $\int \frac{t^2}{9+4t^6}\,dt$ **h)** $\int \frac{1+4t^6}{t^2}\,dt$ **i)** $\int \frac{e^{2x}}{\sqrt{25-e^{4x}}}\,dx$ **j)** $\int \frac{1}{3+12t^2}\,dt$

5. a) If an antiderivative of $f(x)$ is $\sin x - x\sin x + 1$, what was $f(x)$?

b) On a recent test Judy said that $\int 2\cos x \sin x\,dx = \sin^2 x + c$. Elaine said that $\int 2\cos x \sin x\,dx = -\cos^2 x + c$. I gave them both full-credit. How can both be correct if their answers are different? Did I make a mistake?

6. a) Fill in the table for Right(n) for $\int_1^3 3(x^2-1)\,dx$. Be sure to simplify $f(x_i)$.

$f(x)$	$[a,b]$	Δx	x_i	$f(x_i)$	Right(n)

b) If you were to graph $f(x)$ on $[1, 3]$ it would be an increasing function. Is Right(n) an over or under estimate of $\int_1^3 3(x^2-1)\,dx$? Explain.

c) Find $\int_1^3 3(x^2-1)\,dx$ using a limit of Riemann sums. Check your answer by evaluating $\int_1^3 3(x^2-1)\,dx$ using the FTC.

d) Think: On a test once, I asked students to compute Right(100) and Right(200) for $f(x) = e^x$ on an interval $[a, b]$. A student correctly computed both with his calculator but failed to label which is which. Which of her two values is Right(100): 47.6828 or 47.4455? **Explain** clearly how you can tell.

7. Determine these indefinite and definite integrals:

a) $\int 8\sec(4\pi x)\tan(4\pi x),dx$ **b)** $\int \frac{e^{\sqrt{t}}}{\sqrt{t}}\,dt$ **c)** $\int z\tan(6z^2+1)\,dz$ **d)** $\int \sin(\sin t)\cos t\,dt$

e) $\int_0^1 \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$ **f)** $\int_\pi^\pi x^2 \sin^5(x)\,dx$ **g)** $\int \sec(\frac{\pi}{4}x)\,dx$ **h)** $\int \sec^2(\frac{\pi}{4}x)\,dx$

8. The graph of $f(t)$ is given below on the left. Answer the following questions about $F(x) = \int_0^x f(t)\,dt$.

a) On what interval(s) is F increasing? Explain.

b) At what point(s), if any, does F have a local min? Explain.

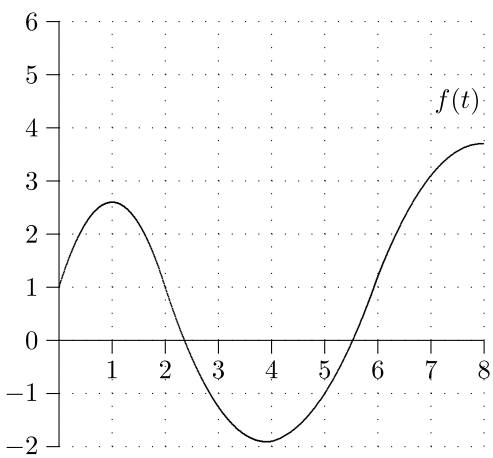
c) On what interval(s) is F concave down? Explain.

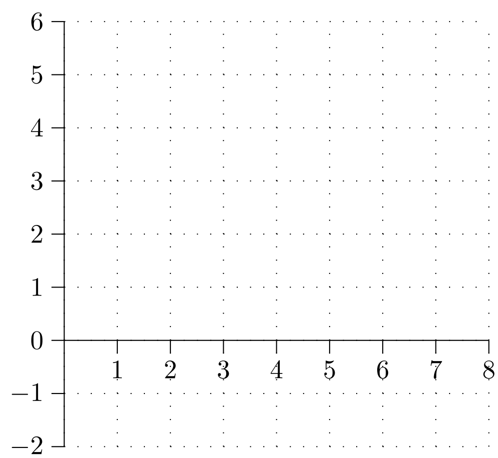
d) Does F have any points of inflection? Explain.

e) Fill in the the table below. Your values should be reasonably accurate.

x	0	1	2	3	4	5	6	7	8
$F(x) = \int_0^x f(t)\,dt$									

f) Graph $F(x)$ on the axes on the right using your table values and knowledge about maxs and mins.





9. Think and Interpret: The **velocity** of an object $v(t)$ in m/s is graphed above on the left (same graph as in previous question).

a) When is the object moving backwards? Explain.

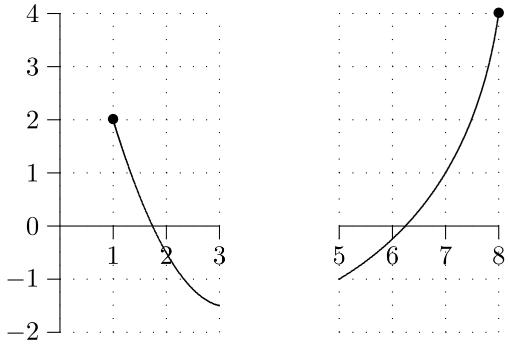
b) Assume that the initial position of the object is $s(0) = 0$. Fill in the row in the table for the position $s(t)$ for the object. Then graph the position function on the axes above on the right.

t	0	1	2	3	4	5	6	7	8
Position $s(t)$									
Distance Traveled $= \int_0^b \lvert v(t)\rvert\,dt$									

c) What is v_{ave} on the interval $[0, 8]$? (Hint: Use the v_{ave} formula and your work above.) At what time(s) t did v_{ave} occur (i.e., when was $v(t)$ actually equal to v_{ave}? Explain.

d) Speed is $\lvert v(t)\rvert$. Graph the speed function for the object on the axes on the left. Distance traveled is the integral of the speed: $\int_0^b \lvert v(t)\rvert\,dt$. Fill in the row in the table above for the distance traveled by the object.

10. When making up this question, the printer jammed and only part of the graph of a differentiable function $F(x)$ was printed out, as shown below. Nonetheless, the graph still provides enough information for you to precisely evaluate $\int_1^8 F'(x)\,dx$. What is the value of this integral. (Look carefully at the integrand.)



← **Graph of $F(x)$.**

11. a) Suppose that the acceleration of an object is $a(t) = e^{2t}$ and that $v(0) = 2$ and $s(0) = 1$, where v and s have their usual meaning. Find $s(t)$.

b) Suppose instead that $a(t) = 24t^2 + 12t$ and that $s(0) = 4$ and $s(1) = 8$. Find $s(t)$.

12. Evaluate the following expression:

$$\frac{d}{dx}\int_{x^2}^{2} 4t^3 e^t\,dt$$

13. On Mars the acceleration due to gravity equals 0.39 that of earth or −12.5 ft/sec/sec. Suppose that a Martian calculus student throws his calculus text *upward* at 25 ft/sec off the roof of a building.

a) If it takes 6 seconds to hit the ground, how high is the building?

b) What was the velocity of the book when it the ground?

14. Let $f(x) = x^2 - x$ on the interval $[1, 3]$. Suppose we want to find Right(n) for this function.

a) Determine the expressions for Δx, x_k, and $f(x_k)$. Be sure to simplify where possible.

b) Determine and simplify the expression for Right(n).

c) Evaluate $\int_1^3 x^2 - x\,dx$ as a limit of Riemann sums.

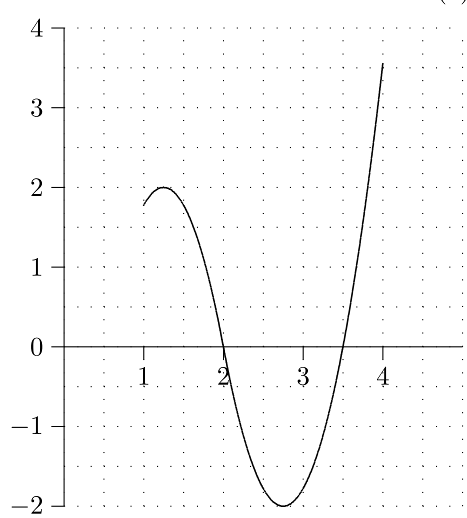
d) Check your answer by using the Fundamental Theorem of Calculus.

15. One of the integral properties states that if we switch the limits of integration, the sign of the integral is switched:

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx.$$

Give a brief explanation of why this should be so. Hint: Think about Δx.

16. Use the graph f below to draw and then estimate the **left**-hand sum Left(6) for the given function on $[1, 4]$.



17. Three similar integrals.

a) $\displaystyle\int \frac{1+x^4}{x}\,dx$ **b)** $\displaystyle\int \frac{x}{1+x^2}\,dx$ **c)** $\displaystyle\int \frac{x^2}{1+x^6}\,dx$

18. My Honda Accord accelerates from 0 to 88 ft/sec (60 mph) in 13 seconds. Assume that acceleration is a constant, a.

a) Find the velocity function of the car (using the velocities at the two times you can eliminate any constants from this function).

b) How far does it travel in this 13 second period?

19. A balloon, rising vertically with a velocity of 8 ft/s, releases a sandbag at the instant it is 64 ft above the ground.

a) How many seconds after its release will the bag strike the ground? Remember $a(t) = -32$ ft/s^2.

b) At what velocity does it hit the ground?

20. **a)** Find the average value of $f(t) = \frac{t}{1+3t^2}$ on $[0, 2]$.

b) Find the average value of $f(t) = 2t^{999} + \pi t^{1001} - 6t^{47} - 17t^5 + \cos(\pi t)$ on $[-1/2, 1/2]$. Why is this EZ?

21. **a)** Page 379 #9. (Answers in text.)

b) Page 379 #23. (Answers in text.)

22. Area between curves: See Assignment from Day 12 and the associated WeBWork problems (and all of the examples on line in the Notes for Day 12).

23. Determine

a) $\displaystyle\int \cos^2(6x)\,dx$ **b)** $\displaystyle\int \sin^2(-2\pi x)\,dx$

Added From Monday's Class

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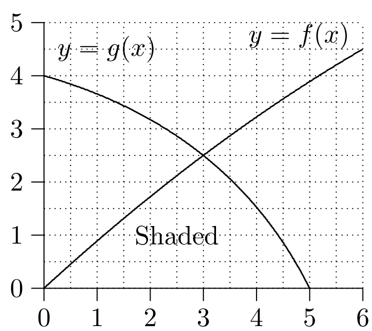
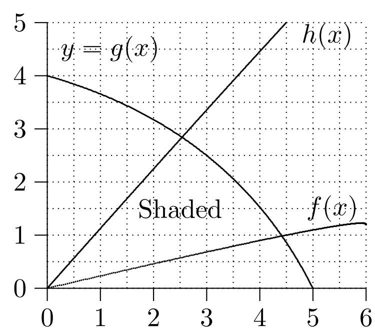
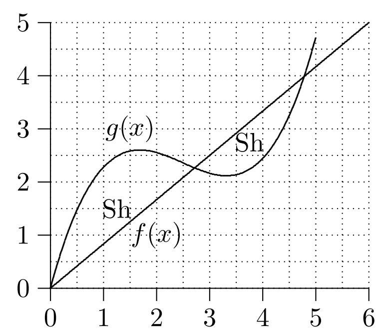
24. Set up the integrals using the functions $f(x)$, $g(x)$, and $h(x)$ and their points of intersection that would be used to find the shaded areas in the three regions below.







25. Sketch the regions for each of the following problems before finding the areas.

a) Find the area enclosed by the curves $y = x^3$ and $y = x^2$. (Answer: 1/12)

b) Find the area enclosed by the curves $y = x^3 + x$ and $y = 3x^2 - x$. (Answer: 1/2)

c) Find the area between the curves $f(x) = \cos x + \sin x$ and $g(x) = \cos x - \sin x$ over $[0, 2\pi]$. (Answer: 8)

Math 131: Answers to Practest 1

1. **a)** Mean Value Theorem: See page 276.

b) FTC II or Mean Value Theorem for Integrals.

c) Partition the interval $[a, b]$ into n equal width subintervals using points

$$a = x_0 < x_1 < x_2 < \cdots < x_n = b.$$

The definite integral of a continuous function f on a closed interval $[a, b]$ is

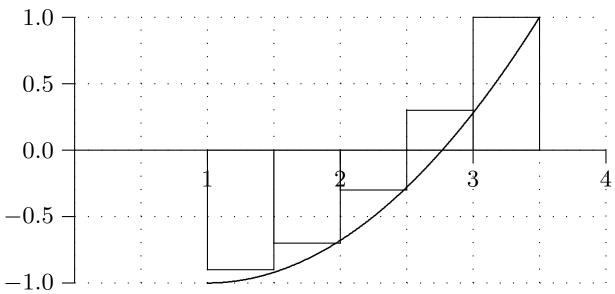
$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x,$$

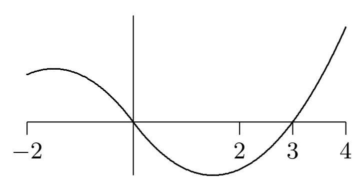
where $\Delta x = \frac{b-a}{n}$ and c_i is some point in the in i-th subinterval.

d) $\frac{d}{dx}\left[\int_{x^3}^{1} \sin^2(t)\,dt\right] = \frac{d}{dx}\left[-\int_{1}^{x^3} \sin^2(t)\,dt\right] = \sin^2(x^3)\cdot 3x^2$, where we used FTC II and the chain rule.

e) FTC II implies $g(x) = \frac{d}{dx}\left[\int_0^x g(t)\,dt\right] = \frac{d}{dx}\left[\frac{\sin x}{e^x}\right] = \frac{e^x \cos x - e^x \sin x}{e^{2x}}$. So $g(0) = \frac{e^0 \cos 0 + e^0 \sin 0}{e^0} = 1$.

2. $\Delta x = \frac{3.5-1}{5} = \frac{1}{2}$. So Right(5) $= (-.9)\cdot\frac{1}{2} + (-0.7)\cdot\frac{1}{2} + (-0.3)\cdot\frac{1}{2} + (0.3)\cdot\frac{1}{2} + (1)\cdot\frac{1}{2} = -0.3$.





3. Use basic integral properties.

a) $\int_3^4 f(x)\,dx = \int_{-2}^4 f(x)\,dx - \int_{-2}^3 f(x)\,dx = 1 - (-1.2) = 2.2$

b) $\int_0^3 f(x)\,dx = \int_{-2}^3 f(x)\,dx - \int_2^0 f(x)\,dx = -1.2 - 3 = -4.2$

c) $\int_{-1}^5 2f(x-1)\,dx = 2\int_{-2}^4 f(x)\,dx = 2$ (horizontal shift)

d) $\int_{-2}^0 f(x) - 4\,dx = 3 + (-4)[0 - (-2)] = -5$

e) $\int_0^2 f(x)\,dx = -\int_{-2}^0 f(x)\,dx = -3$

4. **a)** $u = 4t - 1$, $\frac{1}{4}du = dt$. $\frac{1}{4}\int u^{1/2}\,du = \frac{1}{6}u^{3/2} + c = \frac{1}{6}(4t-1)^{3/2} + c$.

b) Use $\int \frac{a^2}{\sqrt{1-u^2}}\,du = \arcsin\frac{u}{a} + c$ with $a^2 = 25$ so $a = 5$, $u^2 = 4t^2$, so $u = 2t$, $\frac{1}{2}du = dt$. $\frac{1}{2}\int \frac{1}{\sqrt{5^2-u^2}}\,du = \frac{1}{2}\arcsin(\frac{u}{5}) + c = \frac{1}{2}\arcsin(\frac{2t}{5}) + c$.

c) $u = 1 - 4t^2$, $du = -8t\,dt$, $-\frac{1}{8}\,du = t\,dt$. $\int \frac{t}{\sqrt{1-4t^2}}\,dt = -\frac{1}{8}\int \frac{1}{u^{1/2}}\,du = -\frac{1}{4}u^{1/2} + c = -\frac{1}{4}\sqrt{1-4t^2} + c$.

d) Method 1: $u = \sqrt{4-t}$, so $u^2 = 4 - t$, so $u^2 - 4 = t$ so $2u\,du = dt$. $\int t\sqrt{4-t}\,dt = \int (u^2 - 4)\cdot u \cdot 2u\,du = \int 2u^4 - 8u^2\,du = \frac{2}{5}u^5 - \frac{8}{3}u^3 + c = \frac{2}{5}u^5 - \frac{8}{3}u^3 + c = \frac{2}{5}(4-t)^{5/2} - \frac{8}{3}(4-t)^{3/2} + c$

d) Method 2: $u = 4 - t$, so $4 - u = t$ and $-du = dt$. $-\int (4-u)u^{1/2}\,du = -\int 4u^{1/2} - u^{3/2}\,du = -\frac{8}{3}u^{3/2} + \frac{2}{5}u^{5/2} + c = \frac{2}{5}(4-t)^{5/2} - \frac{8}{3}(4-t)^{3/2} + c$.

e) $u^2 = 4t^4$, $u = 2t^2$, $\frac{1}{4}du = t\,dt$. $\frac{1}{4}\int \frac{1}{1+u^2}\,du = \frac{1}{4}\arctan u + c = \frac{1}{4}\arctan(2t^2) + c$.

f) $u = 1 + 4t^4$, $\frac{1}{16}du = t^3 dt$. $\frac{1}{16}\int \frac{1}{u}\,du = \frac{1}{16}\ln|u| + c = \frac{1}{16}\ln(1 + 4t^4) + c$.

g) $a^2 = 9 = 3^2$, $u^2 = 4t^6$, $u = 2t^3$, $\frac{1}{6}du = t^2 dt$. $\frac{1}{6}\int \frac{1}{3^2+u^2}\,du = \frac{1}{6}\cdot\frac{1}{3}\arctan(\frac{u}{3}) + c = \frac{1}{18}\arctan(\frac{2t^3}{3}) + c$.

h) $\int \frac{1+4t^6}{t^2}\,dt = \int t^{-2} + 4t^4\,dt = -t^{-1} + \frac{4}{5}t^5 + c$. (Just divide first by t^2.)

i) Use $\int \frac{a^2}{\sqrt{1-u^2}}\,du = \arcsin\frac{u}{a} + c$ with $a^2 = 25$ so $a = 5$, $u^2 = e^{4x}$, so $u = e^{2x}$, $\frac{1}{2}du = e^{2x}dx$. $\frac{1}{2}\int \frac{1}{\sqrt{5^2-u^2}}\,du = \frac{1}{2}\arcsin(\frac{u}{5}) + c = \frac{1}{2}\arcsin(\frac{e^{2x}}{5}) + c$.

j) $u^2 = 4t^2 \Rightarrow u = 2t \Rightarrow du = 2\,dt \Rightarrow \frac{1}{2}\,du = dt$. So

$$\int \frac{1}{\sqrt{1-4t^2}}\,dt = \frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}\,du = \frac{1}{2}\arcsin u + c = \frac{1}{2}\arcsin 2t + c.$$

k) $\displaystyle\int \frac{1}{3+12t^2}\,dt = \frac{1}{3}\int \frac{1}{1+4t^2}\,dt$. Use $u^2 = 4t^2 \Rightarrow u = 2t \Rightarrow du = 2\,dt \Rightarrow \frac{1}{2}\,du = dt$. So

$$\frac{1}{3}\int \frac{1}{1+4t^2}\,dt = \frac{1}{3}\cdot\frac{1}{2}\int \frac{1}{1+u^2}\,du = \frac{1}{6}\arctan u + c = \frac{1}{6}\arctan 2t + c.$$

5. a) Since $\int f(x)\,dx = \sin x - x\sin x + 1$, then $f(x) = \frac{d}{dx}(\sin x - x\sin x + 1) = \cos x - \sin x - x\cos x$.

b) No. Just take the derivatives of each: $\frac{d}{dx}(\sin^2 x + c) = 2\sin x\cos x$ while $\frac{d}{dx}(-\cos^2 x + c) = -2\cos x(-\sin x) = 2\sin x\cos x$ also. The two antidervatives differ by a constant since $\sin^2 x = -\cos^2 x + 1$.

6. a)

$f(x)$	$[a,b]$	Δx	x_i	$f(x_i)$	Right(n)
$3(x^2-1)$	$[1,3]$	$\frac{2}{n}$	$1+\frac{2i}{n}$	$3\left(\frac{4i^2}{n^2}+\frac{4i}{n}\right)$	$\displaystyle\sum_{i=1}^{n} 3\left(\frac{4i^2}{n^2}+\frac{4i}{n}\right)\left(\frac{2}{n}\right)$

b) Since the function is increasing ($f'(x) = 6x > 0$ on $[1,3]$), Right(n) is an overestimate of $\int_1^3 3x^2 - 3\,dx$ because $f(x_i)$ will be greater than any value of $f(x)$ in $[x_{i-1}, x_i]$

c) $\displaystyle \text{Right}(n) = 3\sum_{i=1}^{n}\left[\frac{4i^2}{n^2}+\frac{4i}{n}\right]\cdot\frac{2}{n} = \frac{24}{n^3}\sum_{i=1}^{n} i^2 + \frac{24}{n^2}\sum_{i=1}^{n} i = \frac{24}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right] + \frac{24}{n^2}\left[\frac{n(n+1)}{2}\right]$

$$= 4\left[\frac{2n^2+3n+1}{n^2}\right] + \frac{12n+12}{n} = 8 + \frac{12}{n} + \frac{4}{n^2} + 12 + \frac{12}{n} = 20 + \frac{24}{n} + \frac{4}{n^2}$$

So $\displaystyle\int_1^3 3(x^2-1)\,dx = \lim_{n\to\infty} \text{Right}(n) = 20$ and using the FTC, $\displaystyle\int_1^3 3x^2 - 3\,dx = x^3 - 3x\Big|_1^3 = (27-9) - (1-3) = 20.$

d) Since e^x is an increasing function Right(n) is an overestimate. Since the estimates improve (smaller overestimate) as n gets larger, then Right$(100) >$ Right(200). So Right$(100) = 47.6828$.

7. a) $\frac{8}{4\pi}\sec(4\pi x) + c = \frac{2}{\pi}\sec(4\pi x) + c$. (Mental adustment)

b) $u = \sqrt{t}$, $2\,du = \frac{1}{\sqrt{t}}\,dt$. $2\int e^u\,du = 2e^u + c = 2e^{\sqrt{t}} + c$.

c) $u = 6z^2 + 1$, $du = 12z\,dz$, $\frac{1}{12}\,du = z\,dz$. So $\int z\tan(6z^2+1)\,dz = \frac{1}{12}\int \tan u\,du = \frac{1}{12}\ln|\sec u| + c = \frac{1}{12}\ln|\sec(6z^2+1)| + c$.

d) $u = \sin t$, $du = \cos t\,dt$. $\int \sin u\,du = -\cos(u) + c = -\cos(\sin t) + c$.

e) $u = e^x + e^{-x}$, $du = (e^x - e^{-x})dx$. Change the limits: When $x = 0$, $u = e^0 + e^0 = 2$; when $x = 1$, $u = e + e^{-1}$. So

$$\int_0^1 \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx = \int_2^{e+e^{-1}} \frac{1}{u}\,du = \ln|u|\Big|_2^{e+e^{-1}} = \ln|e + e^{-1}| - \ln 2.$$

f) $\displaystyle\int_{\pi\leftarrow}^{\pi\leftarrow} x^2\sin^5(x)\,dx = 0$. Integral property.

g) $\int \sec(\frac{\pi}{4}x)\,dx = \frac{4}{\pi}\ln|\sec(\frac{\pi}{4}x) + \tan\sec(\frac{\pi}{4}x)| + c$. "Adjust!"

h) $\int \sec^2(\frac{\pi}{4}x)\,dx = \frac{4}{\pi}\tan(\frac{\pi}{4}x) + c$. "Adjust!"

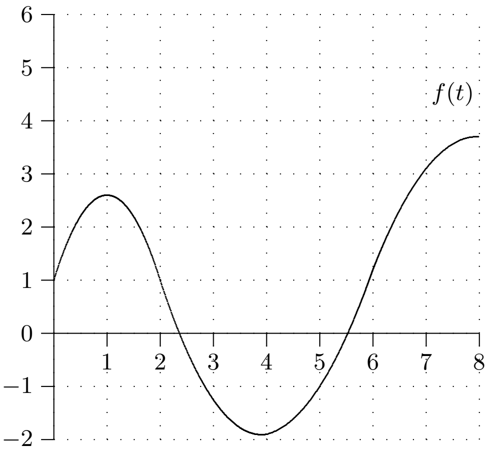
8. The graph of $f(x)$ is given below on the left. Answer the following questions about $F(x) = \int f(x)\,dx$.

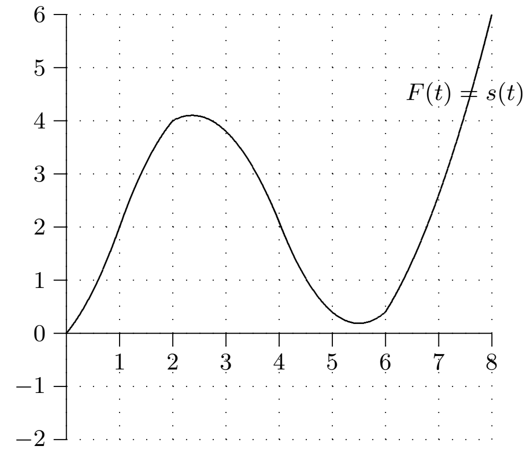
a) F increasaing, so $F' = f > 0$: $[0, 2.5]$ and $[5.5, 8]$.

b) Local min when $F' = f$ changes from negative to positive: $x = 5.5$

c) $\int_0^b f(t)\,dt$ is just the net area under the curve from 0 to b.

b	0	1	2	3	4	5	6	7	8
$F(b) = \int_0^b f(t)\,dt$	0	2	4	3.8	2.1	0.4	0.4	2.6	6





9. **a)** When is the object moving backwards? $v < 0$ so $[2, 5, 5.5]$.

b) Negative acceleration when $a = v' < 0$: so $[1, 4]$.

c) Position is the net area under the curve (same table as above).

t	0	1	2	3	4	5	6	7	8
Position $s(t)$	0	2	4	3.8	2.1	0.4	0.4	2.6	6
Distance Traveled	0	2	4	4.8	6.5	8.2	9	11.2	14.6

d) $v_{\text{ave}} = \frac{1}{8-0}\int_0^8 v(t)\,dt = \frac{1}{8}[s(8) - s(0)] = \frac{1}{8}(6-0) = \frac{3}{4}$. v_{ave} occurred at $c = 2.2$ and 5.8. [Use the graph of $v(t) = f(t)$.]

10. From the graph we see that $F(1) = 2$ and $F(8) = 4$. So $\int_1^8 F'(x)\,dx = F(x)\Big|_1^8 = F(8) - F(1) = 4 - 2 = 2$.

11. **a)** Integrate twice: $v(t) = \int e^{2t}\,dt = \frac{1}{2}e^{2t} + c$. $v(0) = 2 = \frac{1}{2} + c \Rightarrow c = \frac{3}{2}$. $s(t) = \int \frac{1}{2}e^{2t} + \frac{3}{2}\,dt = \frac{1}{4}e^{2t} + \frac{3}{2}t + c$. $s(0) = 1 = \frac{1}{4} + c \Rightarrow c = \frac{3}{4}$. So $s(t) = \frac{1}{4}e^{2t} + \frac{3}{2}t + \frac{3}{4}$.

b) $v(t) = \int 24t^2 + 12t\,dt = 8t^3 + 6t^2 + c$. So $s(t) = \int 8t^3 + 6t^2 + c\,dt = 2t^4 + 2t^3 + ct + d$. Now $s(0) = 4 = d$. So $s(1) = 8 = 2 + 2 + c + 4 \Rightarrow c = 0$. So $s(t) = 2t^4 + 2t^3 + 4$.

12. By FTC II $\displaystyle \frac{d}{dx}\int_{x^2}^{2} 4t^3e^t\,dt = \frac{d}{dx}\left(-\int_2^{x^2} 4t^3e^t\,dt\right) = -4(x^2)^3e^{x^2}\cdot 2x = -8x^7e^{x^2}$.

13. **a)** Given $s''(t) = -12.5$, $s'(0) = 25$. Find $s(0)$. Then $s'(t) = \int -12.5\,dt = -12.5t + c$. But $s'(0) = 25 = -12.5(0) + c$ means $c = 25$ and $s'(t) = -12.5t + 25$. Next, $s(t) = \int -12.5t + 25\,dt = -6.25t^2 + 25t + d$. Now set $s(6) = 0 = -6.25(6)^2 + 25(6) + d \to d = 75$ ft. So $s(t) = -6.25t^2 + 25t + 75$. And $s(0) = 75$.

b) Find $s'(6) = -12.5(6) + 25 = -50$ ft/s.

14. Let $f(x) = x^2 - x$ on the interval $[1, 3]$. Suppose we want to find Right(n) for this function.

a) $\Delta x = \frac{2}{n}$, $x_i = 1 + \frac{2i}{n}$, and $f(x_i) = (1 + \frac{2i}{n})^2 - (1 + \frac{2i}{n}) = 1 + \frac{4i}{n} + \frac{4i^2}{n^2} - (1 + \frac{2i}{n}) = \frac{2i}{n} + \frac{4i^2}{n^2}$. Be sure to simplify where possible.

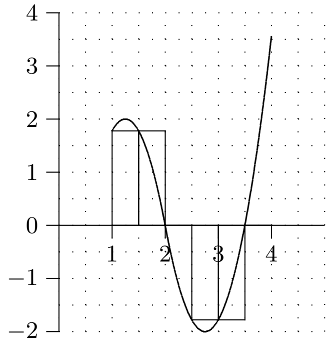
b) So $\displaystyle \text{Right}(n) = \sum_{i=1}^{n} 3\left(\frac{4i^2}{n^2} + \frac{2i}{n}\right)\left(\frac{2}{n}\right)$.

c)
$$\text{Right}(n) = \sum_{i=1}^{n}\left[\frac{4i^2}{n^2} + \frac{2i}{n}\right]\cdot\frac{2}{n} = \frac{8}{n^3}\sum_{i=1}^{n} i^2 + \frac{4}{n^2}\sum_{i=1}^{n} i = \frac{8}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right] + \frac{4}{n^2}\left[\frac{n(n+1)}{2}\right]$$
$$= \frac{4}{3}\left[\frac{2n^2 + 3n + 1}{n^2}\right] + \frac{2n+2}{n} = \frac{8}{3} + \frac{4}{n} + \frac{4}{3n^2} + 2 + \frac{2}{n} = \frac{14}{3} + \frac{6}{n} + \frac{4}{3n^2}$$

So $\int_1^3 x^2 - x\,dx = \lim_{n\to\infty}\text{Right}(n) = \frac{14}{3}$.

d) By FTC, $\int_1^3 x^2 - x\,dx = \frac{1}{3}x^3 - \frac{1}{2}x^2\Big|_1^3 = \left(9 - \frac{9}{2}\right) - \left(\frac{1}{3} - \frac{1}{2}\right) = \frac{14}{3}$.

15. For $\int_b^a f(x)\,dx$, in the Riemann sum $\Delta x = \frac{a-b}{n}$ which is the negative of the Δx for the Riemann sum for $\int_a^b f(x)\,dx$ while the values of $f(x_k)$ remain the same. Thus the sums will be the negatives of each other.

16. 

Note $\Delta x = \frac{4-1}{6} = \frac{1}{2}$. So
$L_6 = (1.8)\frac{1}{2} + (1.8)\frac{1}{2} + (0)\frac{1}{2} + (-1.8)\frac{1}{2} + (-1.8)\frac{1}{2} + (0)\frac{1}{2} = 0.$

17. Similar integrals done differently and one extra.

a) $\displaystyle\int \frac{1+x^4}{x}\,dx = \int \frac{1}{x} + x^3\,dx = \ln|x| + \frac{1}{4}x^4 + c$

b) $u = 1 + x^2 \Rightarrow \frac{1}{2}du = x dx \qquad \int \frac{x}{1+x^2}\,dx = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + c = \frac{1}{2}\ln(1+x^2) + c$

c) $u^2 = x^6 \Rightarrow u = x^3 \Rightarrow \frac{1}{3}du = x^2 dx \qquad \int \frac{x^2}{1+x^6}\,dx = \frac{1}{3}\int \frac{1}{1+u^2}\,du = \frac{1}{3}\arctan u + c = \frac{1}{3}\arctan(x^3) + c$

18. Given: $a(t) = a$ constant. $v_0 = 0$ ft/s. $s_0 = 0$. And $v(13) = 88$ ft/s.

a) For constant acceleration: $v(t) = at + v_0 = at$. But $v(13) = a \cdot 13 = 88 \Rightarrow a = \frac{88}{13}$. So $v(t) = \frac{88}{13}t$.

b) For constant acceleration: $s(t) = \frac{1}{2}at^2 + v_0 t + s_0 = \frac{44}{13}t^2$. Consequently, $s(13) = \frac{44}{13}(13)^2 = 572$ ft.

19. Given: $a(t) = -32$ ft/s^2 is constant. $v_0 = 8$ ft/s. $s_0 = 64$ ft. So check that

- $v(t) = -32t + 8$ ft/s
- and $s(t) = -16t^2 + 8t + 64$ ft.

a) The bag hits the ground when the position $s(t) = -16t^2 + 8t + 64 = 0$. Using the quadratic formula the solution is $t = \frac{1+\sqrt{65}}{4}$ (not $\frac{1-\sqrt{65}}{4}$).

b) The velocity when it hits the ground is $v(\frac{1+\sqrt{65}}{4}) = -32(\frac{1+\sqrt{65}}{4}) + 8 = -8\sqrt{65} \approx 64.5$ ft/s.

20. a) $f_{\text{ave}} = \frac{1}{2-0}\int_0^2 \frac{t}{1+3t^2}\,dt$. $u = 1 + 3t^2$, $du = 6t\,dt$. When $x = 0$, $u = 1$; $x = 2$, $u = 13$. So

$$f_{\text{ave}} = \frac{1}{2}\cdot\frac{1}{6}\int_1^{13}\frac{1}{u}\,du = \frac{1}{12}\ln|u|\Big|_1^{13} = \frac{\ln 13}{12}.$$

b) Use symmetry, split into odd and even terms: $f_{\text{ave}} = \dfrac{1}{1/2-(-1/2)}\displaystyle\int_{-1/2}^{1/2} 2t^{999} + \pi t^{1001} - 6t^{47} - 17t^5 + \cos(\pi t)\,dt$

$$= 1\int_{-1/2}^{1/2} 2t^{999} + \pi t^{1001} - 6t^{47} - 17t^5\,dt + 1\int_{-1/2}^{1/2}\cos(\pi t)\,dt = 0 + 2\int_0^2 \cos(\pi t)\,dt = \frac{1}{\pi}\sin\pi t\Big|_0^{1/2} = \frac{1}{\pi}(1-0) = \frac{1}{\pi}.$$

23. a) Note the 'mental adjustment.' $\int \cos^2(6x)\,dx = \int \frac{1}{2} + \frac{1}{2}\cos(12x)\,dx = \frac{1}{2}x + \frac{1}{24}\sin(12x) + c$.

b) $\int \sin^2(-2\pi x)\,dx = \int \frac{1}{2} - \frac{1}{2}\cos(-4\pi x)\,dx = \frac{1}{2}x - \frac{1}{8\pi}\sin(4\pi x) + c$.

24. Left: $\int_0^3 f(x)\,dx + \int_3^5 g(x)\,dx$. Middle: $\int_0^{2.5} h(x) - f(x)\,dx + \int_{2.5}^{4.5} g(x) - f(x)\,dx$. Right: $\int_0^{2.7} g(x) - f(x)\,dx + \int_{2.7}^{4.7} f(x) - g(x)\,dx$.

25. Sketch the curves! I have only done the integrals here.

a) $A = \int_0^1 x^2 - x^3\,dx = \frac{1}{3}x^3 - \frac{1}{4}x^4\Big|_0^1 = (\frac{1}{3} - \frac{1}{4}) - 0 = \frac{1}{12}$.

b) $A_1 = \int_0^1 (x^3+x) - (3x^2-x)\,dx = \int_0^1 x^3 - 3x^2 + 2x\,dx = \frac{1}{4}x^4 - x^3 + x^2\Big|_0^1 = (\frac{1}{4} - 1 + 1) - 0 = \frac{1}{4}$. $A_2 = \int_1^2 (3x^2-x) - (x^3+x)\,dx = -\frac{1}{4}x^4 + x^3 - x^2\Big|_1^2 = (-4+8-4) - (-\frac{1}{4} + 1 - 1) = \frac{1}{4}$. $A_1 + A_2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$.

c) $A_1 = \int_0^\pi (\cos x + \sin x) - (\cos x - \sin x)\,dx = \int_0^\pi 2\sin x\,dx = -2\cos x\Big|_0^\pi = 2 - (-2) = 4$. $A_2 = \int_\pi^{2\pi} (\cos x - \sin x) - (\cos x + \sin x)\,dx = \int_\pi^{2\pi} -2\sin x\,dx = 2\cos x\Big|_\pi^{2\pi} = 2 - (-2) = 4$. Total area is 8.