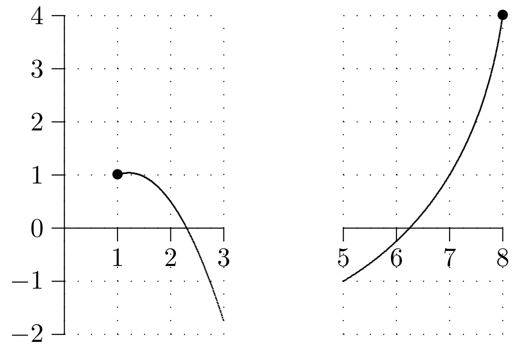
# Math 131 Semi-Final Review

**Warning**: The material from the last week or so on Taylor Polynomials, Power Series, and Taylor/MacLaurin Series is not covered here. I will post additional problems. **Extra Credit**: Be the first to find typos in the questions or answers.

**1.** Do the series questions on Lab 14 for additional series problems beyond those included below.

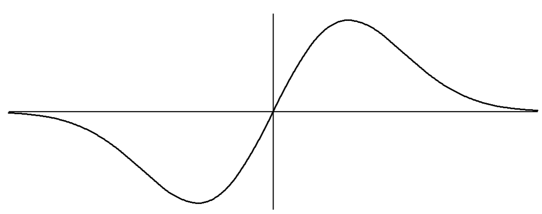
**2.** When making up this test, the printer jammed and only part of the graph of a differentiable function $F(x)$ was printed out, as shown below. Nonetheless, the graph still provides enough information for you to precisely evaluate $\int_1^8 F'(x)\,dx$. What is the value of this integral. (Look carefully at the integrand.)




← **Graph of $F(x)$.**

**3.** Let $f$ be the function whose graph is given below. Use the information in the table, properties of the integral, and the **shape** of $f$ to evaluate the given integrals.

**a)** $\displaystyle\int_3^0 f(x)\,dx$ **b)** $\displaystyle\int_1^4 5+2f(x)\,dx$ **c)** $\displaystyle\int_{-4}^4 f(x)+3\,dx$ **d)** $\displaystyle\int_{-1}^2 f(x)\,dx$



| |
|---|
| $\int_0^1 f(x)\,dx = 0.4$ |
| $\int_0^2 f(x)\,dx = 0.8$ |
| $\int_0^3 f(x)\,dx = 0.9$ |
| $\int_0^4 f(x)\,dx = 1.0$ |

**4.** Review all three previous Practests.

**5.** Find the arc length of $\ln(\cos x)$ on the interval $[0, \pi/3]$. Ans: $\ln|2+\sqrt{3}|$

**6.** Determine whether the alternating series $\displaystyle\sum_{n=1}^{\infty}(-1)^n\left(\frac{\sqrt{n}}{n-1}\right)$ converges. Carefully check whether $a_{n+1} \le a_n$.

**7.** Integral Mix Up: First classify each by the technique that you think will apply: substitution, parts, parts twice, or ordinary methods. (Trig sub covered elsewhere.)

**a)** $\displaystyle\int 2e^{-3x}\,dx$ **b)** $\displaystyle\int \sec^2 x e^{\tan x}\,dx$ **c)** $\displaystyle\int e^x \cos x\,dx$

**d)** $\displaystyle\int x\sec^2 x\,dx$ **e)** $\displaystyle\int \cos(2\pi x)\,dx$ **f)** $\displaystyle\int \cos^2(\pi x)\,dx$

**g)** $\displaystyle\int (2x^2+x)e^x\,dx$ **h)** $\displaystyle\int \arctan x\,dx$ **i)** $\displaystyle\int x^2\ln x\,dx$

**j)** $\displaystyle\int x\sqrt{x-1}\,dx$ **k)** $\displaystyle\int \sec 3x\,dx$ **l)** $\displaystyle\int \sec^3 2x\,dx$

**m)** Censored **n)** $\displaystyle\int \frac{x}{25+x^2}\,dx$ **o)** $\displaystyle\int \frac{1}{1+25x^2}\,dx$

**p)** $\displaystyle\int \cos^3 2x\,dx$ **q)** $\displaystyle\int \tan^4 \pi x\,dx$ **r)** $\displaystyle\int \sin^2 2\pi x\,dx$

**s)** $\displaystyle\int \frac{1}{\sqrt{1-25x^2}}\,dx$ **t)** $\displaystyle\int \frac{\cos x}{\sqrt{1-\sin^2 x}}\,dx$ **u)** $\displaystyle\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}\,dx$

**8.** **a)** Determine $\int (\ln x)^2\, dx$.

**b)** Let $R$ be the region enclosed by $y = \ln x$, the $x$-axis, and $x = e$ in the first quadrant. Rotate $R$ about the $x$-axis and find the volume.

**9.** **a)** Assume that $n$ is a positive integer. Find $\int x^n \ln x\, dx$.

**b)** Find $\int \ln(\sqrt[n]{x})\, dx$.

**10.** Let $R$ be the two-part region enclosed by $y = \cos \pi x$, the $x$-axis, $x = 0$, and $x = 1$. Rotate $R$ about the $x$-axis. Find the volume of the resulting solid. (Ans: $\pi/2$)

**11.** Let $R$ be the region enclosed by $y = \cos x$, the $x$-axis, $x = 0$, and $x = \pi/2$ in the first quadrant. Rotate $R$ about the $y$-axis. Find the volume of the resulting solid using shells. (Ans: $\pi^2 - 2\pi$)

**12.** Evaluate these limits.

**a)** $\lim_{x\to 0} \frac{e^x - \cos x}{2x^3 + 2x}$ **b)** $\lim_{x\to\infty} \frac{x^2 + 7x}{9x^2 + 1}$ **c)** $\lim_{x\to 0} \frac{\cos x + \sin x}{3x + 2}$ **d)** $\lim_{x\to 0} \frac{x \cos x}{x^2 + 2x}$

**e)** $\lim_{x\to\infty} \frac{x \ln x}{e^x}$ **f)** $\lim_{x\to 0} \frac{\cos 2x - \cos 4x}{x^2}$ **g)** $\lim_{x\to 0} \frac{\arctan x}{\sin 5x}$ **h)** $\lim_{x\to 0^+} 2x \ln x$

**i)** $\lim_{x\to\infty} x^2 e^{-x}$ **j)** $\lim_{x\to\infty} \ln(2x + 9) - \ln(x + 7)$ **k)** $\lim_{x\to\infty} (1 + \frac{2}{x})^x$ **l)** $\lim_{n\to\infty} \sqrt[n]{n}$

**m)** $\lim_{n\to\infty} (1 + 3n)^{1/n^2}$

**13.** Try the following.

**a)** $\int_0^\infty \frac{4}{4 + x^2}\, dx$ **b)** $\int_3^\infty \frac{4}{4 - x^2}\, dx$ **c)** Censored **d)** $\int_3^\infty \frac{4x}{4 - x^2}\, dx$

**e)** $\int \frac{4}{\sqrt{4 + x^2}}\, dx$ **f)** $\int \frac{4}{\sqrt{4 - x^2}}\, dx$ **g)** $\int \frac{4x}{(4 + x^2)^{3/2}}\, dx$ **h)** $\int \frac{4x^2}{\sqrt{4 - x^2}}\, dx$

**i)** $\int_0^2 \sqrt{4 - x^2}\, dx$ **j)** $\int \frac{4x + 1}{x^2 - 5x + 4}\, dx$ **k)** $\int \frac{4x}{\sqrt{x - 4}}\, dx$ **l)** $\int \frac{4x + 8}{x^2 + 4x + 5}\, dx$

**m)** $\int \frac{-4x + 4}{(x - 2)^2 x}\, dx$ **n)** $\int_2^4 \frac{\sqrt{x^2 - 4}}{x}\, dx$ **o)** Dropped **p)** $\int \frac{4}{(4 - x)^{2/3}}\, dx$

**q)** $\int \frac{4}{(4 - x^2)^{3/2}}\, dx$ **r)** $\int \sin^3 \pi x\, dx$ **s)** $\int \cos^3 x \sin^2 x\, dx$ **t)** $\int \frac{-5x - 3}{x^2 - 3x}\, dx$

**u)** $\int \frac{8x + 4}{x^3 + x^2 - 2x}\, dx$ **v)** $\int \frac{4x^2 + 8x + 2}{x(x + 1)^2}$ **w)** $\int \sin^2 x + \cos^2 x\, dx$

**14.** Find the average value of $f(x) = \frac{2}{x^2+12x+35}$ on $[-1, 1]$. (Ans: $\frac{1}{2}(2\ln 6 - \ln 4 - \ln 8)$.)

**15.** Find the limit of each of these sequences, if it exists.

**a)** $\left\{\frac{3 + 2\sqrt{n}}{1 + \sqrt{n}}\right\}_{n=1}^\infty$ **b)** $\left\{\frac{3 + 2n}{1 + \sqrt{n}}\right\}_{n=1}^\infty$ **c)** $\{2\arctan(n + 2)\}_{n=1}^\infty$ **d)** $\{\ln(2n) - \ln(n + 1)\}_{n=1}^\infty$

**16.** Does the series $\sum_{n=1}^{\infty} n e^{-n}$ converge? Explain.

**17.** **a)** Does the series $\sum_{n=1}^{\infty} \frac{1}{n^2 + 5n + 6}$ converge?

**b)** Do it again using another test.

**c)** List two other tests that could also be used.

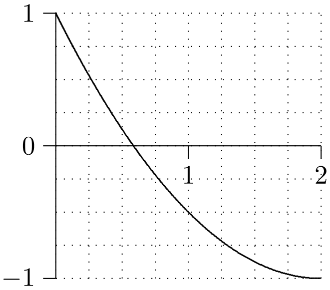
**18.** **a)** Carefully state the Mean Value Theorem and draw a figure that illustrates it.

**b)** Name two instances where we used the Mean Value Theorem this term!

**19.** Know the summation formula for $\sum_{k=1}^{n} k$ and $\sum_{k=1}^{n} k^2$

**20. a)** From a Final Exam: Draw and then estimate Right(4) for the graph of $f$ on $[0, 2]$ below. Be careful about how you draw your rectangles. Watch the scale. How many rectangles should you draw?

**b)** Is your estimate an over-estimate or an underestimate? Explain why.




**21.** Suppose that $f(x) = x^3 - 2x$ on $[0, 2]$.

**a)** Compute Right($n$) for this situation.

**b)** Use your Riemann sum to find $\int_0^2 x^3 - 2x\,dx$. Then check your answer by using antidifferentiation.

**22.** Assume $f$ and $g$ are continuous and that $\int_{-4}^{6} f(x)\,dx = 27$, $\int_0^2 f(x)\,dx = 12$, $\int_2^6 f(x)\,dx = 20$, and $\int_{-4}^{6} g(x)\,dx = -12$. Evaluate the following.

**a)** $\displaystyle\int_0^6 f(x)\,dx$ **b)** $\displaystyle\int_{-4}^0 f(x)\,dx$ **c)** $\displaystyle\int_0^{-4} f(x)\,dx$

**d)** $\displaystyle\int_{-1}^1 f(x+1)\,dx$ **e)** $\displaystyle\int_3^3 x^2 f(x)\,dx$ **f)** $\displaystyle\int_{-4}^6 (f(x) - 4g(x))\,dx$

**23. a)** If $f(x) = \int_1^x \sin(e^t)\,dt$, what is $f'(x)$? What fundamental theorem did you use?

**b)** If $f(x) = \int_x^2 e^{\sin t}\,dt$, what is $f'(x)$?

**c)** Suppose that $x^2 \sin(\pi x) = \int_0^x g(t)\,dt$. Evaluate $g(1)$ and explain your answer. Hint: How can you first find $g(x)$?

**24.** Evaluate $\displaystyle\int_e^{e^2} \frac{1}{t \ln t}\,dt$. (Ans: $\ln 2$.)

**25.** On the final exam for Math 131, Jody says that $\displaystyle\int \cos(\sqrt{x})\,dx = 2\cos(\sqrt{x}) + 2\sqrt{x}\sin(\sqrt{x}) + c$. Determine whether she should receive credit for her answer. Explain. Hint: I do not expect that you can do this integration.

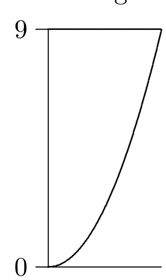
**26.** Find the area of the wedge-shaped region *below* the curves $y = \sqrt{x-1}$, $y = 3 - x$, and above the $x$-axis. Integrate along either axis: your choice! (Ans: 7/6.)

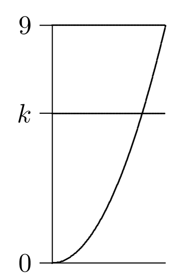
**27.** Find the area enclosed by the curves $y = x^2 + 2x$ and $y = x^3 - 4x$. Draw the figure. (Ans: $21\frac{1}{12}$)

**28.** Find the area of the region bounded by $y = \arcsin x$, the $x$ axis and the line $x = 1$ in the first quadrant. Hint: You can switch axes or do integration by parts. (Ans: $\frac{1}{2}\pi - 1$.)

**29. a)** The region $R$ in the first quadrant enclosed by $y = x^2$, the $y$-axis, and $y = 9$ is shown in the graph on the left below. Find the area of $R$.

**b)** A horizontal line $y = k$ is drawn so that the region $R$ is divided into two pieces of equal area. Find the value of $k$. (See the graph on the right). Hint: It might be easier to integrate along the $y$-axis now.



**30.** **a)** Find the volume of the solid that results when the region enclosed by $y = \sqrt{x}$ and $y = x$ is revolved about the $x$-axis. (Answer: $\pi/6$)

**b)** Find the volume of the solid that results when the region in the first quadrant enclosed by $y = \sqrt{9 - x^2}$, $y = 1$, $y = 3$, and the $y$ axis is revolved about the $y$-axis. (Answer: $28\pi/3$)

**31.** A small canal bouy is formed by taking the region in the first quadrant bounded by the $y$-axis, the parabola $y = 2x^2$, and the line $y = 5 - 3x$ and rotating it about the $y$-axis. (Units are feet.) Find the volume of this bouy. (Answer: $2\pi$ cubic feet.)

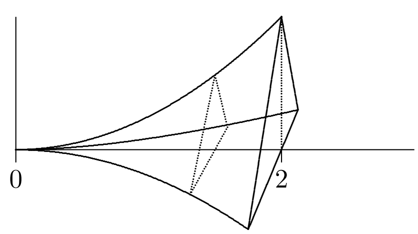
**32.** See Lab 7 if you need practice on Volume Problems.

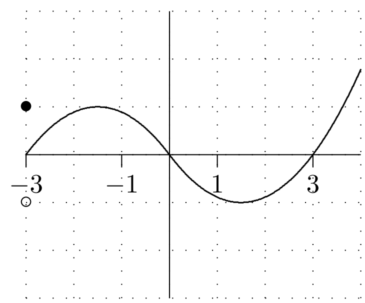
**33.** Find the average value of $f(x) = \dfrac{1}{\sqrt{4 - 36x^2}}$ on the interval $[0, \frac{1}{6}]$. (Ans: $\pi/6$)

**34.** **a)** A tank is formed by rotating the region between $y = x^2$, the $y$-axis and the line $y = 4$ in the first quadrant around the $y$ axis. The oil in the tank has density 50 lbs/ft$^3$. Find the work done pumping the oil to the top of the tank if there is only 1 foot of oil in the tank.

**b)** Suppose the tank is empty and is **filled** from a hole in the bottom to a depth of 3 feet. Find the work done.

**35.** A crystal prism is 2 cm long (below left). Its cross-sections are isosceles triangles whose bases are twice the heights. If the heights are formed by the curve $y = \frac{x^2}{4}$, find the volume of the prism. (Answer: 0.4 cu. cm.)




**36.** Let $F(x)$ be the antiderivative of $f(x)$ on $[-3, 4]$, where $f$ is the function graphed above (right). Since $F$ is an antiderivative of $f$, then $F' = f$. Use this relationship to answer the following questions.

**a)** On what interval(s) is $F$ increasing? Decreasing?

**b)** At what point(s), if any, does $F$ have a local max? Min?

**c)** On what interval(s) is $F$ concave up? Down?

**d)** Does $F$ have any points of inflection?

**e)** Assume $F$ passes through the point $(-3, 1)$ indicated with a $\bullet$; draw a potential graph of $F$.

**f)** Assume, instead, that $F$ passes through $(-3, -1)$ indicated by a $\circ$; draw a graph of $F$. What is the relationship between the two graphs you've drawn?

**37.** Find the interval of convergence for these power series.

**a)** $\displaystyle\sum_{n=0}^{\infty} \frac{n(x-3)^n}{5^{n+1}}$ **b)** $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 3x^n}{n!}$ **c)** $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{3n-2}$ **d)** $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n(x-5)^n}{n^2+1}$

**e)** $\displaystyle\sum_{n=0}^{\infty} (-1)^n n^2 x^n$ **f)** $\displaystyle\sum_{n=0}^{\infty} 9^n x^{2n}$ **g)** $\displaystyle\sum_{n=0}^{\infty} \frac{(n+1)!(x+4)^n}{(2n)!}$ **h)** $\displaystyle\sum_{n=0}^{\infty} \frac{n^n x^n}{n!}$

**38.** Determine whether the following series converge. Justify your answers with an argument.

**a)** $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[7]{n^5+4n}}$ **b)** $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan n}{1+n^2}$ **c)** $\displaystyle\sum_{n=1}^{\infty} \frac{301n}{(n+101)2^n}$ **d)** $\displaystyle\sum_{n=1}^{\infty} \frac{n^6+4}{n^7}$ **e)** $\displaystyle\sum_{n=1}^{\infty} e\left(\frac{\pi}{2!}\right)^{2!n}$

**f)** $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n^2-5n+4}$ **g)** $\displaystyle\sum_{n=1}^{\infty} \sin\frac{1}{n^3}$ **h)** $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{n^2+1}$ **i)** $\displaystyle\sum_{n=1}^{\infty} \frac{5n^2-1}{2n^9+1}$ **j)** $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{7^n+2}$

**39.** Determine whether these alternating series converge conditionally, absolutely, or not at all. **Justify your answers your answers with a complete argument.**

**a)** $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[7]{n^5+4n}}$ **b)** $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(21n+1)}{31n+2}$ **c)** $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{2^n}$ **d)** $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{2n^3+1}}$

## Math 131 Final Review Answers

**1.** See the answers to Lab 14 on line.

**2.** Use antiderivatives: $\int_1^8 F'(x)\,dx = F(x)\big|_1^8 = F(8) - F(1) = 4 - 1 = 3$

**3.** The answers are:

**a)** $-\displaystyle\int_0^3 f(x)\,dx = -0.9$ **b)** $\displaystyle\int_1^4 5\,dx + \int_1^4 f(x)\,dx = 15 + 2(0.6)$ **c)** $\displaystyle\int_{-4}^4 f(x)\,dx + \int_{-4}^4 3\,dx = 0 + 24$

**d)** $= \displaystyle\int_{-1}^0 f(x)\,dx + \int_0^2 f(x)\,dx$ symmetry $= -\displaystyle\int_0^1 f(x)\,dx + \int_0^2 f(x)\,dx = -0.4 + 0.8 = .4$

**5.** $AL = \int_0^{\pi/3} \sqrt{1+\tan^2 x}\,dx = \int_0^{\pi/3} \sec x\,dx = \ln|\sec x + \tan x|\big|_0^{\pi/3}$.

**6.** Use the alternating series test. Check the two conditions.

**a)** $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\sqrt{n}}{n-1} = \lim_{n\to\infty} \sqrt{\frac{n}{(n-1)^2}} = \lim_{n\to\infty} \sqrt{\frac{n}{n^2-2n+1}} = \lim_{n\to\infty} \sqrt{\frac{1/n}{1-2/n+1/n^2}} = 0$ checks.

**b)** $a_{n+1} \le a_n \iff \frac{\sqrt{n+1}}{n} \le \frac{\sqrt{n}}{n-1} \iff (n-1)\sqrt{n+1} \le n\sqrt{n} \iff (n^2-2n+1)(n+1) \le n^2 n \iff n^3 - n^2 - n + 1 \le n^3 \iff 1 \le n^2 + n$ checks.

**7.** Brief Answers to the Mix-Up Problem: (All "$+c$".)

**a)** $-\frac{2}{3}e^{-3x}$ **b)** $e^{\tan x}$ **c)** $\frac{1}{2}e^x(\cos x + \sin x)$
**d)** $x\tan x - \ln|\sec x|$ **e)** $\frac{1}{2\pi}\sin(2\pi x)$ **f)** $\frac{1}{2}x + \frac{1}{4\pi}\sin 2\pi x$
**g)** $(2x^2 - 3x + 3)e^x$ **h)** $x\arctan x - \frac{1}{2}\ln|x^2+1|$ **i)** $\frac{1}{3}x^3\left(\ln x - \frac{1}{3}\right)$
**j)** $\frac{2}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2}$ **k)** $\frac{1}{3}\ln|\sec 3x + \tan 3x|$ **l)** $\frac{1}{4}(\sec 2x\tan 2x + \ln|\sec 2x + \tan 2x|)$
**m)** Censored **n)** $\frac{1}{2}\ln(25 + x^2)$ **o)** $\frac{1}{5}\arctan 5x$
**p)** $\frac{1}{2}\sin 2x - \frac{1}{6}\sin^3 2x$ **q)** $\frac{1}{3\pi}\tan^3 \pi x - \frac{1}{\pi}\tan\pi x + x$ **r)** $\frac{x}{2} - \frac{1}{8\pi}\sin 4\pi x + c$
**s)** $\frac{1}{5}\arcsin 5x$ **t)** $\arcsin(\sin x)$ **u)** $\frac{1}{2}(\arcsin x)^2$

**8. a)** Use parts twice, the first time $u = (\ln x)^2$ and the second time $u = 2\ln x$. Ans: $x(\ln x)^2 - 2x\ln x + 2x + c$.

**b)** From (a): $\int_1^e \pi(\ln x)^2\,dx = \pi\left[x(\ln x)^2 - 2x\ln x + 2x\right]\Big|_1^e = \pi(e-2)$.

**9. a)** $\frac{1}{n+1}x^{n+1}\ln x - \frac{1}{(n+1)^2}x^{n+1} + c$

**b)** $\frac{1}{n}(x\ln x - x)$. Hint: $\ln(\sqrt[n]{x}) = \ln(x^{1/n}) = \frac{1}{n}\ln x$.

**10.** The two pieces are symmetric (so double one of them). Use half-angle formula: $V = 2\cdot\int_0^{1/2}\pi\cos^2\pi x\,dx = 2\pi\int_0^{1/2}\frac{1}{2} + \frac{1}{2}\cos(2\pi x)\,dx = 2\pi\left(\frac{1}{2}x + \frac{1}{4\pi}\sin(2\pi x)\right)\Big|_0^{1/2} = \frac{\pi}{2}$.

**11.** Use parts with $u = x$. $V = \int_0^{\pi/2} 2\pi x\cos x\,dx = 2\pi[x\sin x - \int_0^{\pi/2}\sin x\,dx] = 2\pi[x\sin x + \cos x]\big|_0^{\pi/2}$.

**12.** L'H once for a, d, g. Twice for b, e, f. Convert h and i to quotients. Use log laws for j. Key limits for k and l. For m: take log and convert to quotient.

**a)** 1/2 **b)** 1/9 **c)** 1/2 **d)** 1/2 **e)** 0 **f)** 6
**g)** 1/5 **h)** 0 **i)** 0 **j)** $\ln 2$ **k)** $e^2$ **l)** 1 **m)** 1

**13. a)** $\lim_{b\to\infty}\int_0^b \frac{1}{1+\frac{x^2}{4}}\,dx = \lim_{b\to\infty} 2\arctan\frac{x}{2}\Big|_0^b = \lim_{b\to\infty} 2\arctan\frac{b}{2} - 0 = 2(\frac{\pi}{2}) - 0 = \pi$.

**b)** $\lim_{b\to\infty}\int_3^b \frac{4}{4-x^2}\,dx = \lim_{b\to\infty}\int_3^b \frac{1}{2-x} + \frac{1}{2+x}\,dx = \lim_{b\to\infty}(-\ln|2-x| + \ln|2+x|)\big|_3^b = \lim_{b\to\infty}\ln|\frac{2+x}{2-x}|\big|_3^b = \lim_{b\to\infty}\ln|\frac{2+b}{2-b}| - \ln 5 = \ln 1 - \ln 5 = -\ln 5$.

**d)** $\lim_{b\to\infty}\int_3^b \frac{4x}{4-x^2}\,dx = \lim_{b\to\infty}(-2\ln|4-x^2|)\big|_3^b = \lim_{b\to\infty}\ln|4-b^2| - \ln 5 = +\infty$. Diverges.

**e)** With triangles where $x = 2\tan\theta$: $\int \frac{4}{\sqrt{4+x^2}}\,dx = \int \frac{4}{2\sec^2\theta} 2\sec^2\theta\,d\theta = \int 4\sec\theta\,d\theta = 4\ln|\sec\theta + \tan\theta| + c = 4\ln\left|\frac{\sqrt{4+x^2}+x}{2}\right| + c$

**f)** Similar to the one above. $\int \frac{4}{\sqrt{4-x^2}}\,dx = \int \frac{4}{2\sqrt{1-\frac{1}{4}x^2}}\,dx = 4\arcsin(\frac{x}{2}) + c.$

**g)** With triangles: $\int \frac{8\tan\theta}{8\sec^3\theta} 2\sec^2\theta\,d\theta = \int 2\sin\theta\,d\theta = -2\cos\theta + c = -\frac{4}{\sqrt{4+x^2}}$

**g)** Much easier by $u$-sub: $= \int 2u^{-3/2}\,du = \int -4u^{-1/2} + c = -4(4+x^2)^{-1/2} + c.$

**h)** $\int \frac{16\sin^2\theta}{2\cos\theta} 2\cos\theta\,d\theta = \int 16\sin^2\theta\,d\theta = -8\sin\theta\cos\theta + 8\theta + c = -2x\sqrt{4+x^2} + 8\arcsin(\frac{x}{2}) + c$

**i)** $\int_0^{\pi/2} 2\cos\theta 2\cos\theta\,d\theta = \int_0^{\pi/2} 4\cos^2\theta\,d\theta = 2\cos\theta\sin\theta + 2\theta\Big|_0^{\pi/2} = \pi.$

**j)** $\int \frac{17/3}{x-4} - \frac{5/3}{x-1}\,dx = \frac{17}{3}\ln|x-4| - \frac{5}{3}\ln|x-1| + c.$

**k)** $u = x - 4$. So $\int \frac{4u+16}{u^{1/2}}\,du = \int 4u^{1/2} + 16u^{-1/2}\,du = \frac{8}{3}u^{3/2} + 32u^{1/2} + c = \frac{8}{3}(x-4)^{3/2} + 32\sqrt{x-4} + c.$

**l)** $\int \frac{4x+8}{x^2+4x+5}\,dx = 2\ln|x^2+4x+5| + c.$ (Use $u$-substitution.)

**m)** $\int -\frac{1}{x-2} - \frac{2}{(x-2)^2} + \frac{1}{x}\,dx = -\ln|x-2| + 2(x-2)^{-1} + \ln|x| + c.$

**n)** $\int_0^{\pi/3} \frac{2\tan\theta}{2\sec\theta} 2\sec\theta\tan\theta\,d\theta = 2\int_0^{\pi/3} \tan^2\theta\,d\theta = 2\int_0^{\pi/3} \sec^2\theta - 1\,d\theta = 2\tan\theta - 2\theta\Big|_0^{\pi/3} = 2\sqrt{3} - \frac{2}{3}\pi.$

**o)** Dropped

**p)** $\int -4u^{-2/3}\,du = -12u^{1/3} + c = -12(4-x)^{-1/3} + c.$

**q)** $4\int \frac{2\cos\theta}{8\cos^3\theta}\,d\theta = \int \sec^2\theta\,d\theta = \tan\theta + c = \frac{x}{\sqrt{4-x^2}} + c.$

**r)** Use trig id and then $u$ sub with $u = \cos\pi x$: $= \int \sin\pi x(1-\cos^2\pi x)\,dx = \int \sin\pi x - \cos^2\pi x\sin\pi x\,dx = -\frac{1}{\pi}\cos\pi x + \frac{1}{3\pi}\cos^3(\pi x) + c$

**s)** $= \int \cos x(\cos^2 x)\sin^2 x\,dx = \int \cos x(1-\sin^2 x)\sin^2 x\,dx = \int \cos x(\sin^2 x - \sin^4 x)\,dx$. Use $u = \sin x$. We get $\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + c.$

**t)** Partial Fractions: $\int \frac{8x+4}{x^3+x^2-2x}\,dx = \int \frac{4}{x+1} - \frac{2}{x+2} - \frac{2}{x}\,dx = 4\ln|x+1| - 2\ln|x+2| - 2\ln|x| + c.$

**u)** Partial Fractions: $\int \frac{-5x-3}{x^2-3x}\,dx = \int \frac{1}{x} - \frac{6}{x-3}\,dx = \ln|x| - 6\ln|x-3| + c.$

**v)** $\int \frac{4x^2+8x+2}{x(x+1)^2}\,dx = \int \frac{2}{x} + \frac{2}{x+1} + \frac{2}{(x+1)^2}\,dx = 2\ln|x| + 2\ln|x+1| - 2(x+1)^{-1} + c.$

**w)** $\int \sin^2 x + \cos^2 x\,dx = \int 1\,dx = x + c.$

**14.** Use partial fractions $\frac{1}{1-(-1)}\int_{-1}^{1} \frac{1}{x+5} - \frac{1}{x+7}\,dx = \frac{1}{2}(\ln|x+5| - \ln|x+7|)\Big|_{-1}^{1}.$

**15.** Use L'H for a and b. For $d$ use $\ln\frac{2n}{n+1} = \ln\frac{2}{1+\frac{1}{n}}$.

**a)** 2 **b)** $\infty$ **c)** $\pi$ **d)** $\ln 2$

**16.** Using the ratio test test the series converges because $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{n+1}{e^{n+1}} \cdot \frac{n}{e^n} = \lim_{n\to\infty} \frac{n+1}{n} \cdot \frac{1}{e} = \frac{1}{e} < 1$

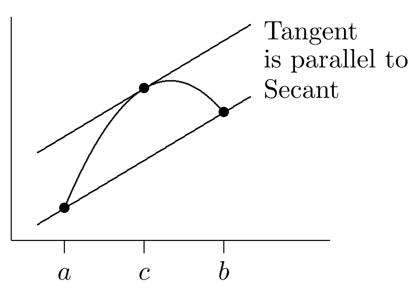
**17. a)** Since $\frac{1}{n^2+5n+6} < \frac{1}{n^2}$ and $\sum \frac{1}{n^2}$ converges by $p$-series test ($p = 2 > 1$), so $\sum \frac{1}{n^2+5n+6}$ by Direct Comparison.

**b)** Use the telescoping sum test (partial fractions): $\sum_{n=1}^{\infty} \frac{1}{n^2+5n+6} = \sum_{n=1}^{\infty} \frac{1}{n+2} - \frac{1}{n+3}$. Show that $s_n = \frac{1}{3} - \frac{1}{n+3}$. So $\lim_{n\to\infty} s_n = \frac{1}{3}$ and so the series converges to $\frac{1}{3}$.

**c)** Use the limit comparison test with $\sum \frac{1}{n^2}$ or use the integral test with partial fractions: $\int_1^{\infty} \frac{1}{x^2+5x+6}\,dx = \lim_{b\to\infty} \ln\left|\frac{x+2}{x+3}\right|\Big|_1^b = -\ln|\frac{3}{4}|$. Converges.

**18. a)** Let $f$ be a continuous function on the closed interval $[a, b]$ and differentiable on the open interval $[a, b]$. Then there is some point $c$ between $a$ and $b$ so that

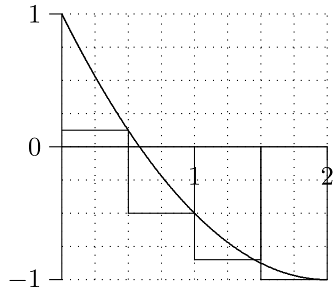
$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$




**b)** In the proof of the Fundamental Theorem of Calculus and the proof of the arc length formula, for example.

**19.** Look them up.

**20.** Right(4) $\approx -1.1$ and it is an underestimate because $f$ is decreasing.




**21.** $\Delta x = \frac{2-0}{n} = \frac{2}{n}$, $x_k = \frac{2k}{n}$, $f(x_k) = (\frac{2k}{n})^3 - \frac{4k}{n}$. Right$(n) \sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n} \left[ \left(\frac{2k}{n}\right)^3 - \frac{4k}{n} \right] \frac{2}{n}$. Now use summation formulas to simplify Right$(n)$.

$$\text{Right}(n) = \frac{2}{n}\sum_{k=1}^{n}\left(\frac{2k}{n}\right)^3 - \frac{2}{n}\sum_{k=1}^{n}\frac{4k}{n} = \frac{16}{n^4}\sum_{k=1}^{n}k^3 - \frac{8}{n^2}\sum_{k=1}^{n}k = \frac{16}{n^4}\left[\frac{n(n+1)}{2}\right]^2 - \frac{8}{n^2}\left[\frac{n(n+1)}{2}\right] = \frac{4(n+1)^2}{n^2} - \frac{4n+4}{n}.$$

Finally, to get the exact "area" evaluate

$$\lim_{n\to\infty} \text{Right}(n) = \lim_{n\to\infty} \frac{4(n+1)^2}{n^2} - \frac{4n+4}{n} = \lim_{n\to\infty} \frac{4n^2+8n+4}{n^2} - \frac{4+\frac{4}{n}}{1} = \lim_{n\to\infty} \frac{4+\frac{8}{n}+\frac{4}{n^2}}{1} - 4 - \frac{4}{n} = 0.$$

**22.** **a)** $\int_0^6 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^6 f(x)\,dx = 12 + 20 = 32.$

**b)** $\int_{-4}^0 f(x)\,dx = \int_{-4}^6 f(x)\,dx - \int_0^6 f(x)\,dx = 27 - 32 = -5.$

**c)** $\int_0^{-4} f(x)\,dx = -\int_{-4}^0 f(x)\,dx = -(-5) = 5.$

**d)** $y = f(x+1)$ is a horizontal shift of $y = f(x)$ one unit to the left, so $\int_{-1}^1 f(x+1)\,dx = \int_0^2 f(x)\,dx = 12.$

**e)** $\int_3^3 x^2 f(x)\,dx = 0$ because the endpoints are the same.

**f)** $\int_{-4}^6 (f(x) - 4g(x))\,dx = \int_{-4}^6 f(x)\,dx - 4\int_{-4}^6 g(x)\,dx = 27 - 4(-12) = 75.$

**23.** **a)** $f'(x) = \sin(e^x)$ by the Fundamental Theorem of Calculus.

**b)** Switch order of limits: then $f'(x) = -e^{\sin x}$

**c)** Take the derivative of each side: Then $\pi x^2 \cos(\pi x) + 2x\sin(\pi x) = g(x)$. So $g(1) = -\pi$.

**24.** Use substitution with $u = \ln t$ and $du = \frac{1}{t}dt$. $x = e \Rightarrow u = 1$ and $x = e^2 \Rightarrow u = 2$. So $\int_1^2 \frac{1}{u}\,du = \ln|u|\Big|_1^2$.

**25.** Jody gets credit. Check that her answer is correct by taking the dervative.

**26.** Use $\int_0^1 \sqrt{x-1}\,dx + \int_1^3 3 - x\,dx = \frac{2}{3}(x-1)^{3/2}\Big|_1^2 + 3x - \frac{1}{2}x^2\Big|_2^3$.

**27.** Use $\int_{-1}^0 (x^3 - 4x) - (x^2 + 2x)\,dx + \int_0^3 (x^2 + 2x) - (x^3 - 4x)\,dx$

**29.** **a)** $A = \int_0^3 9 - x^2\,dx = 9x - \frac{1}{3}x^3\Big|_0^3 = (27 - 9) - 0 = 18.$

**b)** Note: The intersection point of the horizontal line $y = k$ with the parabola is at $(\sqrt{k}, k)$. Find $k$ so that $\frac{A}{2} = \frac{18}{2} = 9 = \int_0^{\sqrt{k}} k - x^2\,dx = kx - \frac{1}{3}x^3\Big|_0^{\sqrt{k}} = k^{3/2} - \frac{1}{3}k^{3/2} = \frac{2}{3}k^{3/2}$. Therefore, $k^{3/2} = \frac{3}{2}\cdot 9 = 13.5 \Rightarrow k = 13.5^{2/3} \approx 5.6696$.

**30.** **a)** Use $\int_0^1 \pi(x - x^2)\,dx$.

**b)** $x = \sqrt{9 - y^2}$. Use disks around $y$-axis. $\int_1^3 \pi(9 - y^2)\,dy$. Or shells $\int_0^{\sqrt{8}} 2\pi x(\sqrt{9 - x^2} - 1)\,dx$.

**31.** $V = \int_0^1 2\pi x[(5 - 3x) - 2x^2]\,dx$.

**33.** $AV = \frac{1}{1/6-0}\int_0^{1/6} \frac{1}{\sqrt{4-36x^2}}\,dx = \frac{1}{1/6}\int_0^{1/6} \frac{1}{\sqrt{a^2-u^2}}\,dx = \frac{1}{6}\cdot\frac{1}{6}\arcsin\frac{6x}{2}\Big|_0^{1/6} = \frac{1}{36}[\arcsin(1/2) - \arcsin(0)] = \pi/36$. Note: $u = 6x$ here so $du = 6dx$ or $\frac{1}{6}du = dx$.

**34.** Use $V = \int_0^2 \frac{1}{2} \frac{x^2}{4} \frac{2x^2}{4}\, dx = \int_0^2 \frac{x^4}{16}\, dx$

**35. a)** $\text{Work} = D \int_a^b A(y)[H-y]\, dy = 50 \int_0^1 \pi(y)[4-y]\, dy = 50\pi \left(2y^2 - \frac{y^3}{3}\right)\Big|_0^1 = 50\pi \left[\left(2 - \frac{1}{3}\right)\right] = \frac{250\pi}{3}$ ft-lbs.

**b)** $\text{Work} = D \int_a^b A(y)[H-y]\, dy = 50 \int_0^3 \pi(y)[y-0]\, dy = 50\pi \left(\frac{y^3}{3}\right)\Big|_0^3 = 50\pi \left[\left(2 - \frac{1}{3}\right)\right] = 450\pi$ ft-lbs.

**36.** Use that $F' = f$ and $F'' = f'$.

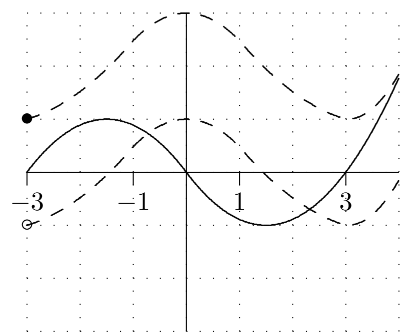
**a)** $F$ increasing means $F' = f > 0$: on $(-3, 0)$ and $(3, 4)$. $F$ decreasing means $F' = f < 0$: on $(0, 3)$.

**b)** $F$ has a local max means $F' = f$ changes from $+$ to $-$: at $x = 0$. $F$ has a local min means $F' = f$ changes from $-$ to $+$: at $x = 3$.

**c)** $F$ concave up means $F'' = f' > 0$, i.e., $f$ is increasing: $(-3, -1.5)$ and $(1.5, 4)$. $F$ concave down means $F'' = f' < 0$, i.e., $f$ is decreasing: $(-1.5, 1.5)$.

**d)** $F$ has a points of inflection when $F'' = f'$ changes sign, i.e., when $f$ changes from increasing to decreasing or *vice versa*: at $x = \pm 1.5$.

**f)** The two graphs are parallel (differ only by a constant).




**37.** In each of these use the ratio test to find the radius of convergence $R$ and then check the two endpoints. These answers are very brief. Yours should be more complete.

**a)** The center $c = 3$. After simplifying $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(n+1)(x-3)}{5n}\right| = \left|\frac{x-3}{5}\right| < 1 \iff |x-3| < 5$. $R = 5$. Check the endpoints. At $c+r = 3+5 = 8$: Simplifies to $\sum_{n=0}^{\infty} \frac{n5^n}{5^{n+1}} = \sum_{n=0}^{\infty} \frac{n}{5}$ diverges by the $n$-th term test. At $c - R = 3 - 5 = -2$: $\sum_{n=0}^{\infty} \frac{n(-5)^n}{5^{n+1}} = \sum_{n=0}^{\infty} \frac{(-1)^n n}{5}$ diverges by the $n$-th term test. Interval: $(-5, 5)$.

**b)** The center $c = 0$. After simplifying $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{x}{n+1}\right| = 0 < 1$. $R = \infty$. Interval: $(-\infty, \infty)$.

**c)** The center $c = 0$. After simplifying $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(3n-2)x}{3n+1}\right| = |x| < 1$. $R = 1$. Check the endpoints. At $c + R = 1$: Using the limit comparison with $\sum \frac{1}{n}$ which diverges by $p$-series test ($p = 1$): $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{1}{3n-2} \cdot \frac{n}{1} = \frac{1}{3}$ so by LCT $\sum_{n=0}^{\infty} \frac{1}{3n-2}$ diverges. At $c - R = -1$: $\sum_{n=0}^{\infty} \frac{(-1)^n}{3n-2}$ converges by the alternating series test. since $\lim_{n\to\infty} \frac{1}{3n-2} = 0$ and $f(x) = (3x-2)^{-1}$ so $f'(x) = (-1)(3x-2)^{-2} < 0$ so the terms are decreasing. Interval: $[-1, 1)$.

**d)** The center $c = 5$. After simplifying $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(n^2+1)(x-5)}{(n+1)^2+1}\right| \overset{\text{HPwrs}}{=} |x - 5| < 1$. $R = 1$. Check the endpoints. At $c + R = 6$: $\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+1}$ converges by the alternating series test since $\lim_{n\to\infty} \frac{1}{n^2+1} = 0$ and $f(x) = (x^2+1)^{-1/2}$ so $f'(x) = (-1/2)(2x)(x^2+1)^{-3/2} < 0$ so the terms are decreasing. At $c - R = 4$: $\sum_{n=0}^{\infty} \frac{1}{n^2+1}$ converges by direct comparison with $\sum \frac{1}{n^2}$: $0 < \frac{1}{n^2+1} < \frac{1}{n^2}$, Since $\sum \frac{1}{n^2}$ converges ($p$-series, $p = 2 > 1$) so does $\sum_{n=0}^{\infty} \frac{1}{n^2+1}$. Interval: $[-1, 1]$.

**e)** After simplifying $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(n^2)x}{(n+1)^2}\right| \overset{\text{HPwrs}}{=} |x| < 1$. $R = 1$. Check the endpoints. At $x = 1$: $\sum_{n=0}^{\infty} (-1)^n n^2$ diverges by the alternating series test $\lim_{n\to\infty} a_n = \lim_{n\to\infty} n^2 \neq 0$. At $x = -1$: $\sum_{n=0}^{\infty} n^2$ diverges by the $n$-th term test again. Interval: $(-1, 1)$.

**f)** The center $c = -4$. After simplifying $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(n+2)(x+4)}{(2n+1)(2n+2)}\right| = \left|\frac{(n+2)x}{4n^2+6n+2}\right| \overset{\text{HPwrs}}{=} 0$. $R = \infty$. Interval: $(-\infty, \infty)$.

**g)** After simplifying $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} |9x^2| < 1 \iff |x^2| < \frac{1}{9} \iff |x| < \frac{1}{3}$. $R = \frac{1}{3}$. Check the endpoints. At $x = \frac{1}{3}$: $\sum_{n=0}^{\infty} 1$ diverges by the $n$-th term test. At $x = -\frac{1}{3}$: $\sum_{n=0}^{\infty} (-1)^n$ diverges by the $n$-th term test. Interval: $(-\frac{1}{3}, \frac{1}{3})$.

**h)** $\sum_{n=1}^{\infty} \frac{n^n x^n}{n!}$ Use the ratio test.

$$\lim_{n\to\infty}\left|\frac{A_{n+1}}{A_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)^{n+1}x^{n+1}}{(n+1)!}\cdot\frac{n!}{n^n x^n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)^{n+1}x}{(n+1)n^n}\right|$$
$$= \lim_{n\to\infty}\left|\frac{(n+1)^n x}{n^n}\right|$$
$$= \lim_{n\to\infty}\left|\left(\frac{n+1}{n}\right)^n\cdot x\right| = \lim_{n\to\infty}\left|\left(1+\frac{1}{n}\right)^n\cdot x\right| = |ex|.$$

By the ratio test, the series *converges* if $|ex| < 1 \iff |x| < \frac{1}{e}$. The radius of convergence is $R = \frac{1}{e}$.

**38. a)** ARGUMENT: $\frac{1}{\sqrt[7]{n^5+4n}}$ and $\frac{1}{n^{5/7}}$ are both positive. Apply the Limit Comparison test.

$$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{1}{\sqrt[7]{n^5+4n}}\cdot\frac{n^{5/7}}{1} \overset{\text{HPwrs}}{=} \lim_{n\to\infty}\frac{n^{5/7}}{\sqrt[7]{n^5}} = 1 > 0.$$

Since $\sum_{n=1}^{\infty}\frac{1}{n^{5/7}}$ diverges by the $p$-series test ($p = \frac{2}{3} \leq 1$), then $\sum_{n=1}^{\infty}\frac{1}{\sqrt[7]{n^5+4n}}$ diverges by the limit comparison test.

**b)** ARGUMENT: Limit comparison test with $\sum_{n=1}^{\infty}\frac{1}{n^2}$. For all $n$ $\frac{\arctan n}{1+n^2}$ and $\frac{1}{n^2}$ are positive. Apply the Limit Comparison test.

$$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{\arctan n}{1+n^2}\cdot\frac{n^2}{1} = \lim_{n\to\infty}\arctan n\cdot\frac{n^2}{n^2+1} = \lim_{n\to\infty}\arctan n\cdot\frac{1}{1+\frac{1}{n^2}} = \frac{\pi}{2} > 0.$$

Since $\sum_{n=1}^{\infty}\frac{1}{n^2}$ converges by the $p$-series test ($p = 2 > 1$), then $\sum_{n=1}^{\infty}\frac{\arctan n}{1+n^2}$ converges by the limit comparison test.

**c)** ARGUMENT: $\frac{301n}{(n+101)2^n}$ and $\frac{1}{2^n}$ are both positive. Apply the Limit Comparison test.

$$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{301n}{(n+101)2^n}\cdot\frac{2^n}{1} = \lim_{n\to\infty}\frac{301n}{n+101} \overset{\text{HPwrs}}{=} \lim_{n\to\infty}\frac{301n}{n} = 301 > 0.$$

Since $\sum_{n=1}^{\infty}\frac{1}{2^n}$ converges (geometric series test, $|r| = \frac{1}{2} < 1$), then $\sum_{n=1}^{\infty}\frac{301n}{(n+101)2^n}$ converges by the limit comparison test.

**d)** ARGUMENT: $\frac{n^6+4}{n^7} = \frac{1}{n} + \frac{4}{n^7} > \frac{1}{n} > 0$ and $\sum\frac{1}{n}$ diverges by the $p$-series test ($p = 1$). So $\sum\frac{n^6+1}{n^7}$ diverges by direct comparison.

**e)** ARGUMENT: Since $\frac{\pi}{2!} = \frac{\pi}{2} > 1$, then $|r| = (\frac{\pi}{2!})^{2!} > 1$. So rewriting the original series as the geometric series $\sum_{n=1}^{\infty} e\left[\left(\frac{\pi}{2!}\right)^{2!}\right]^n$, it diverges.

**f)** ARGUMENT: Limit comparison test with $\sum\frac{1}{n^2}$. For $n \geq 2$ we have $\frac{3}{n^2-3n+4}$ and $\frac{1}{n^2}$ are positive.

$$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{2}{n^2-3n+4}\cdot\frac{n^2}{1} = \lim_{n\to\infty}\frac{2n^2}{n^2-3n+4} \overset{\text{HPwrs}}{=} \lim_{n\to\infty}\frac{2n^2}{n^2} = 2 > 0.$$

Since $\sum\frac{1}{n^2}$ converges by the $p$-series test ($p = 2 > 1$), then $\sum\frac{2}{n^2-3n+4}$ converges by the limit comparison test.

**g)** ARGUMENT: Limit comparison test with $\sum\frac{1}{n^3}$. Since $0 < \frac{1}{n^3} < 1 < \pi/2$, then $\sin\frac{1}{n^3} > 0$.

$$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{\sin\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{x\to\infty}\frac{\sin\frac{1}{x^3}}{\frac{1}{x^3}} \overset{\text{l'Ho}}{=} \lim_{x\to\infty}\frac{\cos(\frac{1}{x^3})(-\frac{3}{x^4})}{-\frac{3}{x^4}} = \lim_{x\to\infty}\cos\frac{1}{x^3} = \cos 0 = 1.$$

Since $\sum\frac{1}{n^3}$ converges by the $p$-series test ($p = 3 > 1$), then $\sum\sin\frac{1}{n^3}$ diverges by the limit comparison test.

**h)** ARGUMENT: Diverges by the $n$th term test.

$$\lim_{n\to\infty}\frac{3^n}{n^2+1} = \lim_{x\to\infty}\frac{3^x}{x^2+1} \overset{\text{l'Ho}}{=} \lim_{x\to\infty}\frac{(\ln 3)3^x}{2x} \overset{\text{l'Ho}}{=} \lim_{x\to\infty}\frac{(\ln 3)^2 3^x}{2} = \infty \neq 0.$$

**i)** ARGUMENT: Limit comparison test with $\sum\frac{1}{n^7}$. Both $\frac{5n^2-1}{2n^9+1}$ and $\frac{1}{n^7} > 0$.

$$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{5^2-1}{2n^9+1}\cdot\frac{n^7}{1} = \lim_{n\to\infty}\frac{5n^9-n^7}{2n^9+1} = \overset{\text{HPwrs}}{=} \lim_{n\to\infty}\frac{5n^9}{2n^9} = \frac{5}{2} > 0.$$

Since $\sum\frac{1}{n^7}$ converges by the $p$-series test ($p = 7 > 1$), then $\sum\frac{5n^2-1}{2n^9+1}$ converges by the limit comparison test.

**j)** ARGUMENT: Direct comparison test with $\sum_{n=0}^{\infty}\frac{2^n}{7^n} = \sum_{n=0}^{\infty}\left(\frac{2}{7}\right)^n$. $0 \leq \frac{2^n}{7^n+2} < \frac{2^n}{7^n} = \left(\frac{2}{3}\right)^n$. But $\sum_{n=0}^{\infty}\left(\frac{2}{3}\right)^n$ converges (geometric series, ($|r| = \frac{2}{7} < 1$), so then $\frac{2^n}{7^n+2}$ converges by the limit comparison test.

**39. a)** ARGUMENT: Check absolute convergence: $\sum_{n=1}^{\infty}\left|\frac{(-1)^n}{\sqrt[7]{n^5+4n}}\right| = \sum_{n=1}^{\infty}\frac{1}{\sqrt[7]{n^5+4n}}$ which diverges by part (a) of the previous problem. Conditional convergence: Use the alternating series test with $a_n = \frac{1}{\sqrt[7]{n^5+4n}} > 0$. Check the two conditions.

1. $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty}\frac{1}{\sqrt[7]{n^5+4n}} = 0$✓.

2. Decreasing? $a_{n+1} \leq a_n \iff \frac{1}{\sqrt[7]{(n+1)^5+4(n+1)}} \leq \frac{1}{\sqrt[7]{n^5+4n}}$ which is true since the denominator of $a_{n+1}$ is greater than for $a_n$ and the numerators are the same.✓By the Alternating Series test, $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt[7]{n^5+4n}}$ converges.

**b)** ARGUMENT: Use the alternating series test with $a_n = \frac{21n+1}{31n+2}$. Check the two conditions.

1. $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty}\frac{21n+1}{31n+2} \overset{\text{HPwrs}}{=} \lim\limits_{n\to\infty}\frac{21n}{31n} = \frac{21}{31} \neq 0$. Fails. Since the first hypothesis is not satisfied, the alternating series test does not apply. In this case the series diverges since the $n$th term does not go to 0.

**c)** ARGUMENT: Check absolute convergence with ratio test: $\lim\limits_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty}\left|\frac{\ln(n+1)}{2^{n+1}}\cdot\frac{2^n}{\ln n}\right| = \lim\limits_{n\to\infty}\left|\frac{\ln(n+1)}{2\ln n}\right| = \lim\limits_{x\to\infty}\left|\frac{\ln(x+1)}{2\ln x}\right| \overset{\text{l'}}{=}$ $\lim\limits_{x\to\infty}\left|\frac{\frac{1}{x+1}}{\frac{2}{x}}\right| = \lim\limits_{x\to\infty}\left|\frac{x}{2(x+1)}\right| \overset{\text{HPwrs}}{=} \lim\limits_{x\to\infty}\left|\frac{x}{2x}\right| = \frac{1}{2} < 1$. The series converges absolutely.

**d)** ARGUMENT: Check absolute convergence: $\sum_{n=1}^{\infty}\left|\frac{(-1)^n}{\sqrt{2n^3+1}}\right| == \sum_{n=1}^{\infty}\frac{1}{\sqrt{2n^3+1}}$. Limit comparison with $\sum\frac{1}{n^{3/2}}$. Both $\frac{1}{\sqrt{2n^3+1}} > 0$ and $\frac{1}{n^{3/2}} > 0$.

$$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{1}{\sqrt{2n^3+1}}\cdot\frac{n^{3/2}}{1} \overset{\text{HPwrs}}{=} \lim_{n\to\infty}\frac{n^{3/2}}{\sqrt{2n^3}} = \frac{1}{\sqrt{2}} > 0.$$

Since $\sum\frac{1}{n^{3/2}}$ converges ($p$-series, $p = \frac{3}{2} > 1$), then $\sum_{n=1}^{\infty}\left|\frac{(-1)^n}{\sqrt{2n^3+1}}\right|$ converges so $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{2n^3+1}}$ converges absolutely.