

1. (22 points) Easy pieces to get you started. (2 points each, unless noted.)

a) Determine $\lim_{n \rightarrow \infty} \left(1 - \frac{4}{n}\right)^{2n}$.

Answer

b) Determine $\int_1^\infty \frac{4}{x^{44}} dx$.

Answer

c) Determine whether $\sum_{n=1}^\infty n^{-4}$ converges. Justify your answer with a brief argument.

Answer

d) Determine $\int \cos^2(4x) dx$.

Answer

e) Determine $\int \sec 4x dx$.

Answer

f) Determine $\int 4^x + \frac{1}{4\sqrt[4]{x^3}} dx$.

Answer

g) Determine $\int x^3 \cos(x^4 + 4) dx$.

Answer

h) Consider the function $f(x) = \frac{1}{x^2}$ on $[1, 4]$. Is the left-hand Riemann sum $\text{Left}(n)$ an overestimate or underestimate of $\int_1^4 \frac{1}{x^2} dx$? Explain in one sentence.

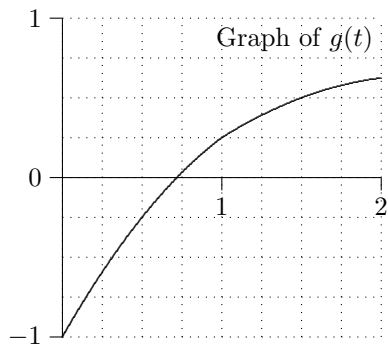
i) Find the sum of the series $\sum_{n=2}^\infty \frac{4}{\sqrt{k+1}} - \frac{4}{\sqrt{k}}$, or explain why it does not converge.

Answer

j) (4 points) Find the sum of the series $\sum_{n=2}^\infty 3 \left(-\frac{4}{5}\right)^n$, or explain why it does not converge.

Answer

2. a) (4 points) Draw and then estimate the **left-hand** Riemann sum $\text{Left}(4)$ for the graph of g on $[0, 2]$ below.



 Answer

- b) (2 points) If $g(t)$ is graphed above and $G(x) = \int_0^x g(t) dt$, on what interval is $G(x)$ decreasing? _____
- c) (8 points) Let $f(x) = 1 + 4x^2$ on the interval $[0, 3]$. Find and simplify the expression for the right-hand endpoint Riemann sum $\text{Right}(n)$. Finally, evaluate $\lim_{n \rightarrow \infty} \text{Right}(n)$. How can you **check your answer**?



Answer

3. Do the following indefinite integrals. Show all work in complete detail.

a) (7 points) $\int \frac{4}{\sqrt{4-x^2}} dx$

Answer

b) (7 points) $\int \frac{4}{x^2\sqrt{x^2-4}} dx$

Answer

c) (7 points) $\int \frac{4}{\sqrt{4+x^2}} dx$

Answer

d) (8 points) $\int (x^2 + x + 1)e^{-x} dx$

Answer

4. a) (8 points) Calculate the arc length of $f(x) = \frac{1}{2}x^2 + 1$ on $[0, 1]$.



Answer

- b) (8 points) Carefully determine $\int_3^5 \frac{4}{x^2 - 4x + 3} dx$.

Answer

- c) (6 points) Carefully evaluate $\int_0^\infty \frac{4}{4 + x^2} dx$.

Answer

5. (15 points) Determine each of the following limits.

a) $\lim_{x \rightarrow 0} \frac{2x - \sin(2x)}{x^2}$

Answer

b) $\lim_{x \rightarrow 0^+} [1 + x^2]^{1/x}$

Answer

c) $\lim_{n \rightarrow \infty} \sqrt[n]{\left(1 + \frac{5}{n}\right)^{4n^2}}$

Answer

d) $\lim_{n \rightarrow \infty} 6 - 2 + \frac{2}{3} - \frac{2}{9} + \frac{2}{27} + \cdots$

Answer

e) $\lim_{n \rightarrow \infty} 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$

Answer

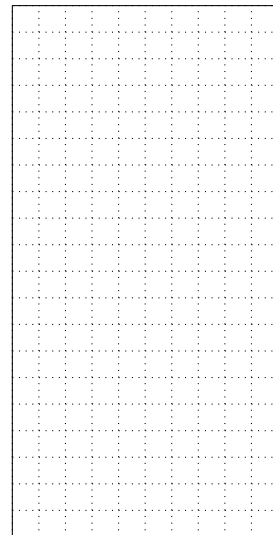
6. a) (10 points) Carefully determine the **interval** of convergence for $\sum_{n=1}^{\infty} \frac{(-1)^n (x-4)^{2n}}{4^n (2n-1)}$. Justify your answer with an argument.

Answer

- b) (5 points) Carefully determine the **radius** of convergence for $\sum_{k=1}^{\infty} \frac{2^k k! x^k}{k^k}$. Justify your answer with an argument.

Answer

7. (10 points) Find the **AREA** enclosed by the curves $f(x) = x^3 - x^2 - 3x + 1$ and $g(x) = 5x^2 + 5x + 1$. (A graph is NOT required.)



Answer

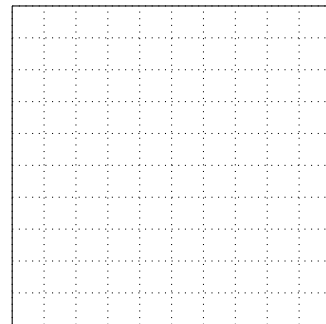
8. a) (10 points) Let R be the region **in the first quadrant** enclosed by the the y -axis, $y = x^3$, and $y = 3x + 2$. Rotate R about the y -axis. Determine the resulting **volume**. (Any method.)

Answer

- b) (5 pts) Let R be the same region as in part (a). Rotate R about the x -axis. **Set up** the integral for the resulting **volume**. (DO NOT EVALUATE THE INTEGRAL.)

Answer

9. (10 points) Let R be the region in the first quadrant enclosed by $y = \frac{1}{4}x^2$, the y -axis, and $y = 4$. Revolve R around the y -axis to form a shallow tank. The tank has oil in it with density of 60 lbs/ft³. If the depth of the oil is 3 feet, calculate the work done in pumping all of the oil to a height 2 feet above the top edge of the tank.



10. (9 points) Determine whether the following **arguments** are correct. Answer ‘correct’ if the argument is completely correct. Answer ‘incorrect’ if there is a mistake in the argument (indicate where the error is) even in the final answer is correct.

a) Using u -substitution, $\int_{-5}^5 \frac{x}{\sqrt{x^2 - 9}} dx = \sqrt{x^2 - 9} \Big|_{-5}^5 = \sqrt{16} - \sqrt{16} = 0$.

b) The series $\sum_{n=1}^{\infty} \frac{1}{5n+2}$ diverges by direct comparison since $0 < \frac{1}{5n+2} < \frac{1}{n}$ and $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by the p -series test.

c) The series $\sum_{n=1}^{\infty} \frac{1}{\arctan(n^2)}$ converges by the n th term test since $\lim_{n \rightarrow \infty} \frac{1}{\arctan(n^2)} = 0$.

11. a) (10 points) Carefully determine whether $\sum_{n=1}^{\infty} \frac{\cos(n\pi)(4n^3 + 1)}{n^4}$ converges absolutely, conditionally, or diverges. Justify your answer with an argument.

Answer

- b) (5 points) Carefully determine whether $\sum_{n=1}^{\infty} \frac{(-1)^n [(n+1)!]^3}{(3n)!}$ converges. Justify your answer with an argument.

Answer

12. a) (6 points) Carefully determine the degree 3 Taylor polynomial $p_3(x)$ for $f(x) = \ln(2+x)$ centered at $x = 1$.

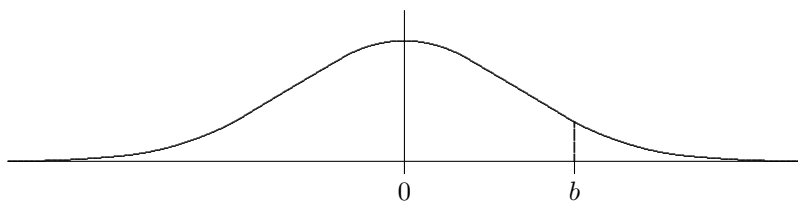
b) (4 points) Now determine the Taylor series for $f(x) = \ln(2+x)$ centered at $x = 1$. Write your answer in summation form.

c) (6 points) Determine the interval of convergence for this series.

 Answer

13. (8 points) Let f be the function whose graph is given below. Use the information in the table, properties of the integral, and the **shape** of f to evaluate the given integrals.

a) $\int_3^0 f(x) dx$ b) $\int_1^4 5 + 2f(x) dx$ c) $\int_{-4}^4 f(x) + 3 dx$ d) $\int_{-1}^2 f(x) dx$



$$\begin{aligned} \int_0^1 f(x) dx &= 0.4 \\ \int_0^2 f(x) dx &= 0.8 \\ \int_0^3 f(x) dx &= 0.9 \\ \int_0^4 f(x) dx &= 1.0 \end{aligned}$$

14. a) (6 points) Determine $\int \cos^6(4x) \sin^3(4x) dx$

b) (8 points) Determine $\int \frac{-4x + 4}{(x - 2)^2 x} dx$

-99. Summation Formula: $\sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}$

-98. Reduction Formulas for Large Powers.

1) $\int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx$

2) $\int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx$

3) $\int \tan^n x \, dx = \frac{1}{n-1} \tan^{n-1} x - \int \tan^{n-2} x \, dx$

4) $\int \sec^n x \, dx = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx$

-97. Guidelines for Products of Tangents and Secants:

a) If the power of secant is *even* and positive, split off $\sec^2 x$.

$$\int \tan^m x \overbrace{\sec^{2k} x}^{n=2k \text{ even}} \, dx = \int \tan^m x \overbrace{(\sec^2 x)^{k-1}}^{\text{convert to tangents}} \overbrace{\sec^2 x}^{\text{split off}} \, dx$$

b) If the power of tangent is odd and positive (and the power of secant is odd), split off $\sec x \tan x$

$$\int \overbrace{\tan^{2k+1} x}^{m=2k+1 \text{ odd}} \sec^n x \, dx = \int \overbrace{(\tan^2 x)^k}^{\text{convert to secants}} \sec^{n-1} x \overbrace{\sec x \tan x}^{\text{split off}} \, dx$$

c) If m is even and n is odd, convert the tangents to secants.

$$\int \overbrace{\tan^{2k} x}^{\text{convert to secants}} \overbrace{\sec^n x}^{n \text{ odd}} \, dx$$

NAME: _____

*Running Total:
Practice Final Exam
Spring 2013*

Problem	Points	Score
1	22	
2	14	
3	29	
4	22	
5	15	
6	15	
7	10	
8	15	
9	10	
10	9	
11	15	
12	21	
13	8	
14	14	
<i>total</i>	219	