Math 135 Homework: Day 08

Reading For The Next Few Classes: Begin Chapter 2 by reading 2.1 and 2.2. Review Sections 1.12–1.15. Read the article by Susanna Epp, a mathematician who teaches at DePaul University. I have given you three questions to answer for Monday based on material in the reading (see below). Also note the following points.

a) In Section 1 she talks briefly about why courses like Math 135 exist.

b) In Section 2, she talks about how complex mathematical statements and reasoning can be. It is not easy. Her students struggle just as you do. Be sure to read the footnote on page 888 that explains some of the Latin terms. **Note:** A rational number r is one that can be written as the ratio of two integers: $r = \frac{m}{n}$ where m and n are integers and $n \neq 0$.. For example, $r = \frac{22}{7}$ is rational, but π is not. This section should seem familiar given your reading in the text.

c) Section 3 discusses the differences between ordinary language and the precision of mathematical language. You will see that you are not alone in struggling with implications and quantifiers. Read actively: Provide the correct interpretations as you read.

d) In Section 6, note that fact that one-on-one discussions between instructor and student have been shown to be very helpful in learning this material, so come and see me!

e) Please read with care the paragraph that begins at the bottom of page 896 and continues to the next page. This is what I mean when I write on your homework, "Explain" what is going on in a truth table.

Prepare for Class Presentation

1. Work problem 7 at the end of Chapter 1 (page 38). I will be looking for volunteers to present this problem **Friday**. We will break it into two parts for two presentations: existence and uniqueness.

2. Also I need a volunteer for Exercise 1.12.1.

Written Assignment Due Monday

These problems are meant to be a simple check on your reading. Problems 1–3 refer to the handout, "The Role of Logic in Teaching Proof." I hope the article gives you a sense of why we ask you to do this course.

1. On page 889, Professor Epp uses an example of everyday language in an implication: "If Tom works overtime, then he's paid extra." She then says that in ordinary conversation, if someone were to dispute this statement, their negation would likely be: "No, If Tom works overtime, then he's not paid extra."

a) State the (mathematically) correct negation.

b) Create a truth table that compares the two negations. Are they different?

c) State the two negations symbolically using only the connectives $\wedge$, $\vee$, and $\sim$. Do not use $\implies$. Are they different?

2. On page 890, Professor Epp uses an example of everyday quantification: "All grass is green." In ordinary conversation, if someone were to dispute this statement, their negation might be any of the following: "Some grass is green" or "Not all grass is green" or "All grass is not green."

a) Which of these are correct negations of the original statement?

b) What is the difference in meaning between those that are correct and those that are not.

c) If you looked at my lawn today, you would see that two-thirds is green and one-third is brown. Which of the negations are true under these circumstances?

3. Do the exercise on page 896: Write "Every polynomial function is continuous" in the form: $\forall$ __________, if __________ then __________.

4. a) Give a direct proof of Exercise 2.2.4 on page 42 of *Chapter Zero*. You will need to work from the definition of subset. We won't have "covered" sets yet, but the material should be familiar.

Class Work: Uniqueness; Direct Proofs

1. **Uniqueness theorems** assert that there is exactly one object with "certain properties" that makes "something happen."

a) For any real number b and any real number $m \neq 1$, there is a unique point of intersection between the lines $y = x$ and $y = mx + b$.

b) If a is a non-zero real number, then a has a unique multiplicative inverse.

c) There is a unique multiplicative identity for the real numbers.

d) There is a unique circle that passes through the three points $(1, 0)$, $(0, 0)$, and $(0, 1)$.

e) Suppose that $f(x)$ is a differentiable function on $(-\infty, \infty)$ and that $f'(x) < 0$ for all x. If a is a real number such that $f(a) = 0$, then, in fact, a is the unique real number so that $f(a) = 0$.

2. Schumacher says that **direct proofs** of theorems of the form "If A, then B" are done by *assuming that the hypothesis is true* and then showing that the conclusion is also true.

a) Why don't we have to consider the case when the hypothesis is false?

b) Most direct proofs (at this early stage) depend on the using the definitions of the terms involved and any previous theorems that you know that apply. Here's an example. *If r is rational, then r^2 is rational.* Give a direct proof.

c) Give a direct proof: If n is an even integer, then $2n^2 + 5n + 3$ is odd.

d) **Definition:** An integer n is divisible by an integer k if and only if $n = km$ where m is also an integer. Prove: If n is an odd integer, then $11n^2 - 7$ is divisible by 4.

Partial Answers to Day 5

1. Remember $P \implies Q$ is logically equivalent to $\sim P \vee Q$.

b) $\sim [(A(x) \wedge B(x)) \implies C(x)] \iff \sim [\sim (A(x) \wedge B(x)) \vee C(x)] \iff (A(x) \wedge B(x)) \wedge \sim C(x)$

c) $\sim [A \iff B] \iff \sim [(A \implies B) \wedge (B \implies A)]$

$$\iff \sim [(\sim A \vee B) \wedge (\sim B \vee A)] \iff [(A \wedge \sim B) \vee (B \wedge \sim A)]$$

d) $\sim (\forall r \exists s \forall y,\ [A(s, y) \implies B(r, y)]) \iff \exists r \forall s \exists y,\ \sim [A(s, y) \implies B(r, y)]$

$$\iff \exists r \forall s \exists y,\ \sim [\sim A(s, y) \vee B(r, y)] \iff \exists r \forall s \exists y,\ A(s, y) \wedge \sim B(r, y)$$

2. Here are some preferred negations suggested by peers.

c) "There is someone that may not attend the concert free" is better as "There is someone who must pay to attend the concert."

d) "Everyone doesn't play golf on Friday afternoons" is better as "No one plays golf on Friday afternoons."

e) "None of the children has clean hands" means $\forall x, \sim C(x)$ so the negation is $\exists x, C(x)$ or "There is a child with clean hands." Another way: "None of the children has clean hands" means $\sim \exists x, C(x)$ so the negation is again $\exists x, C(x)$.

g) "All lines through P are not parallel to ℓ" could be better expressed as "No line through P is parallel to ℓ" or even "All lines through P intersect ℓ."

h) Remember the implicit "for all" quantification in most implications. "A triangle is equilateral and it is not isosceles" really should include the existential quantifier "There is a triangle that is equilateral and it is not isosceles." [Symbolically, $\forall x,\ A(x) \implies B(x)$ which is the same as $\forall x,\ \sim A(x) \vee B(x)$. So the negation is $\exists x,\ A(x) \wedge \sim B(x)$ or in words "There is an equilateral triangle that is not isosceles." This shows that to prove the original statement false, all we need to do is find a single counterexample.]

i) Finally, the negation of "at most one" is "at least two" or "more than one." So we might write: "There exists an increasing function f and f has two or more roots."