

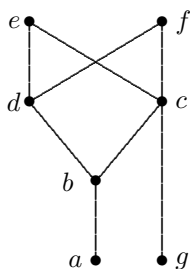
Math 135 Homework: Day 28

Reading, Etc.

You should be able to define each of the following terms: $A \times B$, relation on A , reflexive, symmetric, anti-symmetric, transitive, equivalence relation, partial order, total order, minimal (maximal) element, least (greatest) element, immediate predecessor (successor), upper (lower) bound, least upper (greatest lower) bound.

Class Work

- Find all lattice diagrams for four elements.
 - How many different partial orders are there on a set with four elements?
- Let A, \leq be a partially ordered set. If x is a greatest element of A , then x is a maximal element of A .
- Let A, \leq be a partially ordered set. Show by example that immediate predecessors (successors) need not be unique.
 - We know \mathbb{R} with ordinary \leq is a partially ordered set. What is (are) the immediate successor(s) of 1.2? What is (are) its immediate predecessor(s)?
 - $A = \{1, 2, 3, 4\}$ under \subseteq is a partially ordered set. What is (are) the immediate successor(s) of $\{1, 2\}$? What is (are) its immediate predecessor(s)?
 - \mathbb{N} under ordinary \leq is a partially ordered set. What is (are) the immediate successor(s) of 2? What is (are) its immediate predecessor(s)? What is the immediate predecessor of 1? Are there any elements of \mathbb{N} that do not have immediate successors?
- Let A, \leq be a partially ordered set and let K be a non-empty subset of A . Define the terms **lower bound** for K , **greatest lower bound** for K , and **greatest element** of K .
- Do Exercise 4.2.21 on page 77.
- What are the elements of D_{30} ?
 - Let $K = \{1, 3, 5\} \subseteq D_{30}$. What are the upper bounds for K ? Least upper bound? Greatest element of K ? Lower bounds? Greatest lower bound? Least element of K ?
 - Same question as (b) for $K = \{10, 15\}$.
- Let $A = \{a, b, c, d, e, f, g\}$. For each subset K_i below, what are the upper bounds for K_i ? Least upper bound? Greatest element of K_i ? Lower bounds? Greatest lower bound? Least element of K_i ?



- $K_1 = \{b, c, d\}$
- $K_2 = \{c, d\}$
- $K_3 = \{c, d\}$
- $K_4 = \{b, g\}$
- $K_5 = \{d, e, f\}$

Presentation: For Friday or Monday

- Let A, \leq be a **totally** ordered set. If x is a maximal element of A , then x is a greatest element of A . (Similarly for minimal elements.)
- Let A, \leq be a **totally** ordered set. If y is an immediate successor of x , then y is unique. (Similarly for immediate predecessors.)
- Let A, \leq be a partially ordered set and let K be a non-empty subset of A . If K has a least upper bound x , then it is unique. (Similarly for greatest lower bound.)
- Let $A = \{a, b, c\}$. How many relations on A are reflexive? Justify your answer.
 - Suppose that A contains n elements. How many relations on A are reflexive? (Your answer will be a function of n .)