Proofs Involving Sets

Recall these definitions (Quiz on Monday on them.)

DEFINITION 4.4.1. Let A and B be two sets.

1. A is a **subset** of B (denoted $A \subseteq B$) if for all $x \in A$ we have $x \in B$.
2. A and B are **equal** (denoted $A = B$) if $A \subseteq B$ and $B \subseteq A$. That is if for all $x \in A$ we have $x \in B$ and for all $y \in B$ we have $y \in A$.
3. A is a **proper subset** of B (denoted$A \subset B$) if $A \subseteq B$ and $A \neq B$. In other words, if for all $x \in A$ we have $x \in B$ and there exists $y \in B$ such that $y \notin A$.
4. $A \cup B = \{x : x \in A \text{ or } x \in B\}$ is the **union** of the sets A and B.
5. $A \cap B = \{x : x \in A \text{ and } x \in B\}$ is the **intersection** of the sets A and B.
6. The **(set) difference** $A - B$ is the set $\{x : x \in A \text{ and } x \notin B\}$. This is also the **relative complement** of B in A.
7. Let U be a universal set and $A \subseteq U$. We define the **complement** of A in U by

$$\overline{A} = U - A = \{x \in U \text{ and } x \notin A\}.$$

Basics of Set Proofs

1. Definition: X is a **subset** of Y if

Prove: $X \subseteq Y$.		
Method of Proof	Assumption/First Step of the Proof	Goal (What you must show)
Element chase (Subset)		

2. Definition: X is a **proper subset** of Y if

To prove $X \subset Y$ we must prove two things,

1.

2.

Prove: $X \subset Y$.		
Method of Proof	Assumption/First Step of the Proof	Goal (What you must show)

☞ The second part of a proper subset proof is an example of "an existence proof." Propose a particular candidate $y \in Y$ and show that $y \notin X$. How you get y is scrapwork, and NOT part of the the proof.

3. Definition: X **equals** Y if

To prove $X = Y$ we must prove two things,

1.

2.

Prove: $X = Y$**.**		
Method of Proof	Assumption/First Step of the Proof	Goal (What you must show)

4. Subset proof Problem A. Suppose that $Y = \{x \in \mathbb{R} : x^2 + 10 > 135\}$ and $Z = \{x \in \mathbb{R} : 2x^2 + 7 \geq 150\}$. Prove $Y \subseteq Z$.

Prove: $Y \subseteq Z$**.**		
Method of Proof	Assumption or first step of the Proof	Goal (What you must show/do in the proof)

(*a*) Scrap work for proving: $Y \subseteq Z$.

(*b*) Prove: $Y \subseteq Z$.

5. **Subset proofs.** Suppose that $A = \{x \in \mathbb{R} : 2x + 1 \geq 9\}$ and $B = \{x \in \mathbb{R} : x^3 - 10 \geq 54\}$.

Prove: $A = B$.		
Method of Proof	Assumption or first step of the Proof	Goal (What you must show/do in the proof)
Part 1 of Proof		
Part 2 of Proof		

(*a*) Scrap work for proving: $A = B$.

(*b*) Prove: $A = B$.

6. Subset proofs. In this problem $\mathcal{F}$ represents the set of all functions that are defined on the interval $[0, 1]$ (for example, the functions you used in calculus).
Let $C = \{f(x) = 2ax + b \in \mathcal{F} : a + b = 1;\ a, b \in \mathbb{R}\}$ and $D = \left\{g(x) \in \mathcal{F} : \int_0^1 g(x)\,dx \leq 1\right\}$.

Prove: $C \subset D$.		
Method of Proof	Assumption or first step of the Proof	Goal (What you must show/do in the proof)
Part 1		
Part 2		

(*a*) Find examples of each of the following:

A function in C: $f(x) =$

A function NOT in C: $F(x) =$

A function in D: $g(x) =$

A function NOT in D: $G(x) =$

(*b*) Scrap work for proving: $C \subset D$.

(*c*) Prove: $C \subset D$.