Math 204 Day 2 p10-11

#1

$$\begin{array}{l} 2x+y=1 \\ 6x+3y=b \end{array} \rightarrow \begin{bmatrix} 2 & 1 & 1 \\ 6 & 3 & b \end{bmatrix} \xrightarrow{R_2-3R_1 \to R_2} \begin{bmatrix} 2 & 1 & 1 \\ 0 & 0 & b-3 \end{bmatrix} \rightarrow \begin{array}{l} 2x+y=1 \\ 0=b-3 \end{array}$$

a) The system will be inconsistent if $b \neq 3$ because the second equation will not be of the form $0=0$.
3 There will be a pivot in the rightmost column.

b) The system is consistent when $b=3$. In that case we have

$$\begin{array}{l} 2x+y=1 \\ 8=0 \end{array}$$

So y is a free variable. There are an infinite number of solutions in this case. There is never a unique solution.

#2 a) #10

3
$$\begin{bmatrix} 1 & 3 & 0 & -2 & -7 \\ 0 & 1 & 0 & 3 & 6 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix} \xrightarrow{R_2-3R_4 \to R_2} \begin{bmatrix} 1 & 3 & 0 & -2 & -7 \\ 0 & 1 & 0 & 0 & 12 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix} \xrightarrow{R_1+2R_2 \to R_1} \begin{bmatrix} 1 & 3 & 0 & 0 & -11 \\ 0 & 1 & 0 & 0 & 12 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix}$$

$$\xrightarrow{R_1-3R_2 \to R_1} \begin{bmatrix} 1 & 0 & 0 & 0 & -47 \\ 0 & 1 & 0 & 0 & 12 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix} \quad \begin{array}{l} x_1=-47 \\ x_2=12 \\ x_3=2 \\ x_4=-2 \end{array} \quad \text{or} \quad (-47, 12, 2, -2)$$

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b)
$$\begin{bmatrix} 1 & -5 & 4 & -3 \\ 2 & -7 & 3 & -2 \\ -2 & 1 & 7 & -1 \end{bmatrix} \xrightarrow{R_2-2R_2 \to R} \begin{bmatrix} 1 & -5 & 4 & -3 \\ 0 & 3 & -5 & 4 \\ -2 & 1 & 7 & -1 \end{bmatrix} \xrightarrow{R_3+2R_1 \to R_3} \begin{bmatrix} 1 & -5 & 4 & -3 \\ 0 & 3 & -5 & 4 \\ 0 & -9 & 15 & -7 \end{bmatrix}$$

3

$$\xrightarrow{R_3+3R_2 \to R_3} \begin{bmatrix} 1 & -5 & 4 & -3 \\ 0 & 3 & -5 & 4 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

<u>Inconsistent</u> The final equation becomes $0=5$ (pivot in rightmost column)

c) #4

3
$$\begin{bmatrix} 2 & 0 & -6 & -8 \\ 0 & 1 & 2 & 3 \\ 3 & 6 & -2 & -4 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1 \to R_1} \begin{bmatrix} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 3 & 6 & -2 & -4 \end{bmatrix} \xrightarrow{R_3-3R_1 \to R_3} \begin{bmatrix} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 6 & 7 & 8 \end{bmatrix} \xrightarrow{R_3-6R_2 \to R_3}$$

$$\begin{bmatrix} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & -5 & -10 \end{bmatrix} \xrightarrow{-\frac{1}{5}R_3 \to R_3} \begin{bmatrix} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_2-2R_3 \to R_2} \begin{bmatrix} 1 & 0 & -3 & -4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_1+3R_3 \to R_1} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Solution $\begin{array}{l} x_1=2 \\ x_2=-1 \\ x_3=2 \end{array}$ or $(2,-1,2)$

1d) #16

$$\begin{bmatrix} 2 & 0 & 0 & -4 & -10 \\ 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ -3 & 2 & 3 & 1 & 5 \end{bmatrix} \xrightarrow[\to R_4]{R_4+\frac{3}{2}R_1} \begin{bmatrix} 2 & 0 & 0 & -4 & -10 \\ 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & 2 & 3 & -5 & -10 \end{bmatrix} \xrightarrow[\to R_4]{R_4-\frac{2}{3}R_2} \begin{bmatrix} 2 & 0 & 0 & -4 & -10 \\ 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & 0 & 1 & -5 & -10 \end{bmatrix}$$

3 $\xrightarrow[\to R_4]{R_4-R_3} \begin{bmatrix} 2 & 0 & 0 & -4 & -10 \\ 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & 0 & 0 & -9 & -9 \end{bmatrix}$ The system is now in echelon form. There is no pivot in the final column so the system is consistent.

$(-3, 5, -5, 1)$

e) #18 Is there a solution to the system? This corresponds to a point of intersection.

$$\begin{bmatrix} 2 & 4 & 4 & 4 \\ 0 & 1 & -2 & -2 \\ 2 & 3 & 0 & 0 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1 \to R_1} \begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & 1 & -2 & -2 \\ 2 & 3 & 0 & 0 \end{bmatrix} \xrightarrow[\to R_3]{R_3-2R_1} \begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & 1 & -2 & -2 \\ 0 & -1 & -4 & -4 \end{bmatrix}$$

3 $\xrightarrow[\to R_3]{R_3+R_2} \begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & -6 & -6 \end{bmatrix} \xrightarrow[\to R_3]{\frac{1}{6}R_3} \begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \leftarrow$ R_2+2R_3, R_1-2R_3

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ $x_1=0$, $x_2=0$, $x_3=1$ $(0,0,1)$ There is exactly 1 solution — so there is a unique point of intersection.

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f) $\begin{bmatrix} 1 & h & -5 \\ 2 & -8 & 6 \end{bmatrix} \xrightarrow[\to R_2]{R_2-2R_1} \begin{bmatrix} 1 & h & -5 \\ 0 & -8-2h & 16 \end{bmatrix} \xrightarrow{-\frac{1}{2}R_2} \begin{bmatrix} 1 & h & -5 \\ 0 & 4+h & -8 \end{bmatrix}$

3 To avoid a pivot in the final column (inconsistent), we need $4+h \neq 0$. The system <u>is</u> consistent as long as we do not have $0=-8$ in the final equation. So $4+h \neq 0$ or $\boxed{h \neq -4}$

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g) $\begin{bmatrix} -4 & 12 & h \\ 2 & -6 & -3 \end{bmatrix} \xrightarrow[\to R_2]{R_2+\frac{1}{2}R_1} \begin{bmatrix} -4 & 12 & h \\ 0 & 0 & -3+\frac{1}{2}h \end{bmatrix}$

3 To be consistent we must have $0=-3+\frac{1}{2}h$. Otherwise there's a pivot in the final column.

So $\boxed{h=6}$ is the <u>only</u> value for h!

p11 #33 $\qquad\qquad\qquad$ p3

#3 Given

a)
$$T_1 = (10+20+T_2+T_4)/4 \quad \text{or} \quad 4T_1 - T_2 \quad - T_4 = 30$$
$$T_2 = (T_1+20+40+T_3)/4 \qquad -T_1 + 4T_2 - T_3 = 60$$
$$T_3 = (T_4+T_2+40+30)/4 \qquad -T_2 + 4T_3 - T_4 = 70$$
$$T_4 = (30+10+T_1+T_3)/4 \qquad -T_1 \quad - T_3 + 4T_4 = 40$$

3

b) Switch Rows 1 and 4 to start

$$R_4 \begin{bmatrix} -1 & 0 & -1 & 4 & 40 \\ -1 & 4 & -1 & 0 & 60 \\ 0 & -1 & 4 & -1 & 70 \\ 4 & -1 & 0 & -1 & 30 \end{bmatrix} \xrightarrow[R_2-R_1 \to R_2]{-R_1 \to R_1 \text{ and}} \begin{bmatrix} 1 & 0 & 1 & -4 & -40 \\ 0 & 4 & 0 & -4 & 20 \\ 0 & -1 & 4 & -1 & 70 \\ 4 & -1 & 0 & -1 & 30 \end{bmatrix} \xrightarrow[R_4-4R_4 \to R_4]{\frac{1}{4}R_2 \to R_2} \begin{bmatrix} 1 & 0 & 1 & 4 & -40 \\ 0 & 1 & 0 & -1 & 5 \\ 0 & -1 & 4 & -1 & 70 \\ 0 & -1 & -4 & 15 & 190 \end{bmatrix}$$

3

$$\begin{matrix} \\ \\ R_3+R_2 \\ R_4+R_2 \end{matrix} \begin{bmatrix} 1 & 0 & 1 & -4 & -40 \\ 0 & 1 & 0 & -1 & 5 \\ 0 & 0 & 4 & -2 & 75 \\ 0 & 0 & -4 & 14 & 195 \end{bmatrix} \xrightarrow{R_4+R_3} \begin{bmatrix} 1 & 0 & 1 & -4 & -40 \\ 0 & 1 & 0 & -1 & 5 \\ 0 & 0 & 4 & -2 & 75 \\ 0 & 0 & 0 & 12 & 270 \end{bmatrix}$$

$$\tfrac{1}{12}R_4 \begin{bmatrix} 1 & 0 & 1 & -4 & -40 \\ 0 & 1 & 0 & -1 & 5 \\ 0 & 0 & 4 & -2 & 75 \\ 0 & 0 & 0 & 1 & 22\tfrac{1}{2} \end{bmatrix} \begin{matrix} R_1+4R_4 \\ R_2+R_4 \\ R_3+2R_4 \\ \\ \end{matrix} \begin{bmatrix} 1 & 0 & 1 & 0 & 50 \\ 0 & 1 & 0 & 0 & 27\tfrac{1}{2} \\ 0 & 0 & 4 & 0 & 120 \\ 0 & 0 & 0 & 1 & 22\tfrac{1}{2} \end{bmatrix} \xrightarrow{\frac{1}{4}R_3} \begin{bmatrix} 1 & 0 & 1 & 0 & 50 \\ 0 & 1 & 0 & 0 & 27\tfrac{1}{2} \\ 0 & 0 & 1 & 0 & 30 \\ 0 & 0 & 0 & 1 & 22\tfrac{1}{2} \end{bmatrix}$$

$$\begin{matrix} R_1-R_3 \\ \to R_1 \\ \\ \\ \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 20 \\ 0 & 1 & 0 & 0 & 27\tfrac{1}{2} \\ 0 & 0 & 1 & 0 & 30 \\ 0 & 0 & 0 & 1 & 22\tfrac{1}{2} \end{bmatrix} \qquad \begin{aligned} T_1 &= 20 \\ T_2 &= 27\tfrac{1}{2} \\ T_3 &= 30 \\ T_4 &= 22\tfrac{1}{2} \end{aligned} \quad \text{or} \quad (20, 27\tfrac{1}{2}, 30, 22\tfrac{1}{2})$$