Assignment 6 (Part 1)

Due Wednesday, February 10th in Class

1. Practice writing proofs. I wanted to have you prove $A(c\mathbf{u}) = c(A\mathbf{u})$, but the proof is done in the text on page 39. Make sure you read it. Instead prove the following two facts.

(*a*) Use the definition of scalar multiplication of vectors to prove: If $\mathbf{v} \in \mathbb{R}^n$, then $0\mathbf{v} = \mathbf{0}_n$, where the 0 on the left a scalar 0 and $\mathbf{0}_n$ on the right is the zero vector in $\mathbb{R}^n$.

Note: This is usually written as $0\mathbf{v} = \mathbf{0}$, without the subscript.

(*b*) Use part(a) and the DEFINITION of matrix-vector multiplication as a linear combination of the columns of A to prove: If A is an $m \times n$ matrix, then $A\mathbf{0}_n = \mathbf{0}_n$, where $\mathbf{0}_n$ on the left side of the equation is in $\mathbb{R}^n$ and $\mathbf{0}_m$ on the right side is in $\mathbb{R}^m$.

Note: This is usually written as $A\mathbf{0} = 0$ without the subscripts.

2. Section 1.5, Exercise 8. Be careful. You are given the *coefficient* matrix, not the augmented matrix for homogeneous system $A\mathbf{x} = \mathbf{0}$.

3. (*a*) Describe all solutions of the **homogeneous** system below on the left in parametric vector form. What type of geometric set does it form in $\mathbb{R}^4$?

$$\begin{aligned} x_1 + 3x_2 - 3x_3 + 7x_4 &= 0 \\ x_2 - 4x_3 + 5x_4 &= 0 \\ -2x_1 - 8x_2 + 14x_3 - 24x_4 &= 0 \end{aligned} \qquad \begin{aligned} x_1 + 3x_2 - 3x_3 + 7x_4 &= 1 \\ x_2 - 4x_3 + 5x_4 &= 2 \\ -2x_1 - 8x_2 + 14x_3 - 24x_4 &= -6 \end{aligned}$$

(*b*) Describe all solutions of the **non-homogeneous** system above on the right. Describe the geometry of the solution set compared to the solution set of part (a).

4. Answer each of the following. These questions make you think about Theorems 1.2 and 1.4 and about what we called Fact 2 in Friday's class.

(*a*) Suppose A is a 5×5 matrix with 4 pivot positions.

1. Must the equation $A\mathbf{x} = \mathbf{0}$ have a nontrivial solution?
2. Must the equation $A\mathbf{x} = \mathbf{b}$ have a solution for EVERY $\mathbf{b} \in \mathbb{R}^5$?

(*b*) Suppose A is a 5×4 matrix with 4 pivot positions.

1. Must the equation $A\mathbf{x} = \mathbf{0}$ have a nontrivial solution?
2. Must the equation $A\mathbf{x} = \mathbf{b}$ have a solution for EVERY $\mathbf{b} \in \mathbb{R}^5$?

(*c*) Suppose A is a 4×5 matrix with 4 pivot positions.

1. Must the equation $A\mathbf{x} = \mathbf{0}$ have a nontrivial solution?
2. Must the equation $A\mathbf{x} = \mathbf{b}$ have a solution for EVERY $\mathbf{b} \in \mathbb{R}^4$?

5. Let $A = \begin{bmatrix} \mathbf{a}_1 \cdots \mathbf{a}_n \end{bmatrix}$ and $B = \begin{bmatrix} \mathbf{b}_1 \cdots \mathbf{b}_n \end{bmatrix}$ both be $m \times n$ matrices. We define the **matrix sum** of A and B by adding the matrices column by column. That is,

$$A + B = \begin{bmatrix} (\mathbf{a}_1 + \mathbf{b}_1) \cdots (\mathbf{a}_n + \mathbf{b}_n) \end{bmatrix}.$$

Prove that matrix addition is commutative, that is, $A + B = B + A$, by using appropriate an vector property of $\mathbb{R}^m$.

6. I will add additional problems on Tuesday or Wednesday that will be due Friday.