Office Hour Help: M & W 2:30–4:00, Tu 2:00–3:30, & F 1:30–2:30 or by appointment. Website: `http://math.hws.edu/~mitchell/Math204S16/index.php`.

☕ *Reading, Practice, and Review*

1. Read Section 1.2 and review Section 1.1.

2. (*a*) You should be able to define these key concepts: **linear system, solution, solution set, equivalent systems,** $m \times n$ **matrix, coefficient matrix, augmented matrix, row operations, and row equivalent matrices.**

(*b*) For next time the key concepts in Section 1.2: **pivot, pivot column, echelon form, reduced row echelon form, basic variables, free variables.**

3. (*a*) **Key Fact about Row Equivalence:** If the augmented matrices of two linear systems are row equivalent, then the two systems have the same solution set.

(*b*) How can you tell from the reduced row echelon form of a system whether it has no, exactly one, or infinitely many solutions?

(*c*) **General Linear System.** In a system of m equations in n unknowns, what does a_{ij} denote? What about b_k?

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n &= b_2 \\
a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots + a_{3n}x_n &= b_2 \\
\vdots \qquad\qquad & \vdots \ \vdots \\
a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_m &= b_m
\end{aligned}
$$

4. Practice (not collected). Relatively easy: Page 10–11 #1, 5, 9, 13, 15, 19, 21, and 23. A bit harder 25 and 27.

Classwork/Practice

(*a*) Determine which of these **augmented** matrices are in echelon form (but not reduced row echelon form), which are in RREF, and which are neither.

(*b*) For those in EF or RREF, what are the pivot columns?

(*c*) Since these are augmented matrices, which systems in EF or RREF are not consistent?

$$
\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}
\quad
\begin{bmatrix} 1 & 0 & 6 & 3 \\ 0 & 1 & -6 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\quad
\begin{bmatrix} 0 & 0 & 1 & 2 & 0 & 0 & -3 & 0 & 7 \\ 1 & 0 & 0 & 0 & 1 & 0 & 6 & 0 & 8 \\ 0 & 0 & 0 & 0 & 0 & 1 & 9 & 0 & 11 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix}
\quad
\begin{bmatrix} 1 & 1 & 1 & 8 & 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 4 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 5 \end{bmatrix}
$$

$$
\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix}
\quad
\begin{bmatrix} 4 & 0 & 2 & 6 \\ 0 & 1 & 9 & 8 \\ 0 & 0 & 0 & 7 \end{bmatrix}
\quad
\begin{bmatrix} 1 & 0 & 6 & 0 & 0 & 4 & 0 & 11 \\ 0 & 8 & 0 & 1 & 0 & 7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 12 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 6 \end{bmatrix}
\quad
\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 4 & 0 & 11 \\ 0 & 1 & 0 & 0 & 0 & 7 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 12 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 6 \end{bmatrix}
$$

$$
\begin{bmatrix} 1 & 10 & 0 & 4 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\quad
\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 4 & 0 & 11 \\ 0 & 1 & 0 & 1 & 0 & 7 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 12 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 6 \end{bmatrix}
\quad
\begin{bmatrix} 4 & 6 & 1 & 1 \\ 0 & 1 & 6 & 6 \\ 0 & 0 & 0 & 9 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\quad
\begin{bmatrix} 1 & 0 & 1 & 4 & 0 & 0 & 4 & 0 & 4 \\ 0 & 1 & 0 & 0 & 1 & 0 & 4 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 8 & 0 & 8 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 9 \end{bmatrix}
$$

Hand In

Hand the following problems on Monday at the beginning of class.

1. Consider the system of linear equations below:

$$\begin{aligned} 2x + y &= 1 \\ 6x + 3y &= b \end{aligned}$$

For what value(s) of b is the system inconsistent? For what values of b does the system have infinitely many solutions? For what value(s) of b does the system have a unique solution? Show your work/justify your answers.

2. Section 1.1, pages 10–11. Do these by hand; do not use Maple (though you may always check your answers with Maple).

(*a*) #10. Continue to reduced row echelon form and determine the solution. Use matrix notation and label each step. Note that each step should consist of a SINGLE elementary row operation.

(*b*) #12. Use matrix notation and label each step. Note that each step should consist of a SINGLE elementary row operation.

(*c*) #16. Use matrix notation and label your steps (you may carefully do more than one at once). You can answer the question from the echelon form. (RREF not required.)

(*d*) #18. How does this question translate into a question about systems of equations? Use matrix notation to solve the relevant system, and show your work/reasoning.

(*e*) #20. Show your work/explain your reasoning.

(*f*) #22. Show your work/explain your reasoning.

3. An application to the physics of heat transfer:

(*a*) Page 11 #33. Show your work as was done for T_1 in the problem statement. Check your answer in the back.

(*b*) Page 11 #34. Your choice: Solve the system in part (a). Either use Maple (print out and interpret the result) or use matrix notation to solve the system. (In the latter case, switch rows 1 and 4 to start).