*Day 11 Homework.*

☞ Review Section 1.8 and read Section 1.9. Key Ideas: Transformation, Linear Transformation, Properties of Linear Transformations (see page 66, (3), (4), (5)), Linear Transformations are Matrix Transformations (Theorem 10), and standard matrix of a linear transformation.

☞ Section 1.8, page 68: #1 3, 5, 7, 9, 15, 17. All of these are indirectly practice for the exam. After reading, try the T/F in #21.

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*Facts and Theorems for Test 1.* Know and be able to use the following theorems.

See your text for the precise statements.

**1.** Uniqueness of Reduced Row-Echelon Form (Theorem 1.1)

**2.** Existence and Uniqueness Theorem (Theorem 1.2)

**3.** Properties of Scalar Multiplication and Vector Addition (p. 27)

**4.** (*a*) Equivalent Representations Theorem (Theorem 1.3) $A\mathbf{x} = \mathbf{b}$ is the same as a vector equation and is solved using the augmented matrix $[a\,\mathbf{b}]$. (p. 36)

(*b*) Corollary: $A\mathbf{x} = \mathbf{b}$ has a solution if and only if $\mathbf{b}$ is a linear combination of the columns of $A$ (if and only if $\mathbf{b}$ is Span $\{\mathbf{a}_1, \ldots \mathbf{a}_n\}$.) (p. 36)

**5.** Spanning and Pivots Theorem: These are equivalent Theorem 1.4 (p. 37) :

(*a*) There are pivots in every ROW of $A$.

(*b*) Columns of $A$ span $\mathbb{R}^m$

(*c*) $A\mathbf{x} = \mathbf{b}$ is consistent for EVERY $\mathbf{b} \in \mathbb{R}^m$.

(*d*) EVERY $\mathbf{b}$ in $\mathbb{R}^m$ is a linear combination of the columns of $A$.

**6.** Properties (Linearity) of Matrix-Vector Multiplication (Theorem 1.5)

**7.** Homogeneous Systems are Consistent (p. 43) and Homogeneous Solutions Theorem (p. 43): ($A\mathbf{x} = \mathbf{0}$ has a non-trivial solution if and only if there is a free variable.)

**8.** Independence of Matrix Columns (p. 57) iff $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

Remember: iff means if and only if.

**9.** Characterization of Dependent Sets. (p.58) (A vector in the set is a linear combination of the others.)

**10.** (*a*) Surplus of Vectors (More Vectors than Entries $\Rightarrow$ linear dependence) (p. 59)

(*b*) Dependence of Sets Containing the Zero Vector. (p. 59)

*Definitions.* Memorize the definitions of these terms for the exam: row equivalent matrices, linear independence (dependence), pivot position, the span of $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$, linear combination, homogeneous and nonhomogeneous system.

See the Glossary in the text.

*Pivot Dictionary.* Results about pivots:

(*a*) For a *particular vector* $\mathbf{b}$: No pivot in rightmost column of *augmented* $[A\ \mathbf{b}] \iff A\mathbf{x} = \mathbf{b}$ is consistent

(*b*) $A$ has a pivot in every row $\iff A\mathbf{x} = \mathbf{b}$ is ALWAYS consistent for all $\mathbf{b} \iff$ the columns of $A$ span $\mathbb{R}^m$

(*c*) The *coefficient matrix* $A$ has a pivot in every column $\iff A\mathbf{x} = \mathbf{0}$ has only the trivial solution $\iff$ the columns of A are independent