☞ Review Section 1.8; read Section 1.9. Key Ideas: Transformation, Linear Transformation, Properties of Linear Transformations (see page 66, (3), (4), (5)), Linear Transformations are Matrix Transformations (Theorem 10). New definitions: **standard matrix** of a linear transformation, **one-to-one** transformation, **onto** transformation.

☞ POSTED ONLINE: Short Assignment 7 due Wednesday. WeBWorK HW4 on lin trans due Sat night.

☞ Practice: (not handed in—check answers in back of text): Section 1.8, page 68: #1 3, 5, 7, 9, 15, 17. Try the T/F in #21. Practice: Section 1.9, page 78: #15–21 odd.

DEFINITION. A transformation (mapping) T is **linear** if

(1) $T(\mathbf{u}+\mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$ for all $\mathbf{u}, \mathbf{v}$ in the domain of T;

(2) $T(c\mathbf{u}) = cT(\mathbf{u})$ for all scalars c and all $\mathbf{u}$ in the domain of T.

NOTE: We say that a *linear transformation preserves the operations of addition and scalar multiplication.*

NOTE: When A is an $n \times m$ matrix, the transformation $T : \mathbb{R}^n \to R^m$ by $T(\mathbf{x}) = A\mathbf{x}$ is linear using basic matrix algebra: $A(\mathbf{u}+\mathbf{v}) = A(\mathbf{u}) + A(\mathbf{v})$ and $A(c\mathbf{u}) = cA(\mathbf{u})$

NOTE: We will see that every linear transformation $T : \mathbb{R}^n \to R^m$ is actually a matrix transformation.

EXERCISE 1.8.1. Determine whether

(1) $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1^2 \\ x_3 \end{bmatrix}$. Always start by setting up notation.

Solution: Check the two properties. Let $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$.

(2) $T : \mathbb{R}^2 \to \mathbb{R}^3$ by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_2 \\ x_1 \\ x_1 + 2x_2 \end{bmatrix}$.

Solution: Check the two properties. Let $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$.

EXERCISE 1.8.2. Determine the matrices A for each of the following linear transformations.

(3) The linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ above.

(4) $T : \mathbb{R}^3 \to \mathbb{R}^2$ by $T(\mathbf{e}_1) = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$, $T(\mathbf{e}_2) = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$, and $T(\mathbf{e}_3) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

(5) Determine $T\left(\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}\right)$ for the transformation in part (4).