Math 204: Day 14

Reading and Practice

Office Hour Help: M & W 2:30–4:00, Tu 2:00–3:30, & F 1:30–2:30 or by appointment. Website: `http://math.hws.edu/~mitchell/Math204S16/index.php`.

1. a) Today we will begin work in Section 2.1. This is basic matrix algebra and I think we can cover much of it quickly. Here are some problems to try over the next few days (not collected). Look ahead in Section 2.2.

b) Key terms and concepts: $n \times n$ identity matrix, $m \times n$ zero matrix, diagonal entries, $n \times n$ diagonal matrix, matrix multiplication (columns of AB are linear combinations of the columns of A), *matrix multiplication is not commutative*, A^T the transpose of A. Next up: multiplicative inverses of square matrices *sometimes* exist.

2. The mechanics of matrix operations. Read about matrix multiplication if we do not get there and try

a) Try page 100–101 #1, 3, 5, 7, 8, 9.

b) Suppose that A is $m \times n$ and that $CA = I_n$. What size is C? Now suppose that $AD = I_m$. What size is D? Compute CAD two ways: as $(CA)D$ and $C(AD)$. So what can you say about about the matrices C and D? What can you say about their sizes? Now what can you say about the size of A?

Hand In Next Friday

Stretching Exercise: Proofs using Definitions. Here are a few problems that will appear on the next hand in. I will be adding more. These review some definitions and properties of linear transformations.

One of the goals of this course is to have you become more adept at 'doing proofs.' The two questions below just reuire using the definitions of the terms involved. When you want to to prove that something is an XYZ, then you need to verify that it has the defining property of an XYZ.

1. One of the concepts we will encounter later in the term is the kernel of a linear transformation. **Definition:** Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be linear. The **kernel** of T is the set of all vectors $\mathbf{x} \in \mathbb{R}^n$ such that $T(\mathbf{x}) = \mathbf{0}$

a) Prove that the kernel of T is closed under addition: that is, if $\mathbf{x}$ and $\mathbf{y}$ are both in the kernel of T, then so is the vector $\mathbf{x} + \mathbf{y}$.

b) Prove that the kernel of T is closed under scalar multiplication: that is, if $\mathbf{x}$ is in the kernel of T, then so is the vector $c\mathbf{x}$ for any scalar c.

2. This problem asks you to think about the definition of range. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be linear. Suppose that $\mathbf{a}$ and $\mathbf{b}$ are both in the range of T.

a) Prove that the vector $\mathbf{a}+\mathbf{b}$ is also in the range of T. (This shows that the range of T is closed under addition.)

b) You know the next question: Is the range of T closed under scalar multiplication: that is, if $\mathbf{a}$ is in the range of T, then so is the vector $c\mathbf{a}$ for any scalar c.

3. This is not hard; you just have to be careful. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ and $S : \mathbb{R}^m \to \mathbb{R}^p$ both be linear transformations. (Note the sizes!) Define the composite transformation by $G : \mathbb{R}^n \to \mathbb{R}^p$ by $G(\mathbf{x}) = S(T(\mathbf{x}))$. Prove that G is a linear transformation. (Check the properties!)