Day 15: Reading and Practice

Office Hour Help: M & W 2:30–4:00, Tu 2:00–3:30, & F 1:30–2:30 or by appointment. Website: `http://math.hws.edu/~mitchell/Math204S16/index.php`.

1. Read Section 2.2 and Review Section 2.1 which we should finish next time. This is basic matrix algebra and I think we can cover much of it quickly.

(*a*) Key terms and concepts: $n \times n$ identity matrix, $m \times n$ zero matrix, diagonal entries, $n \times n$ diagonal matrix, matrix multiplication (columns of AB are linear combinations of the columns of A), *matrix multiplication is not commutative*, A^T the transpose of A. Next up: multiplicative inverses of square matrices *sometimes* exist.

2. Practice. Not to be handed in, check the answers in the back.

(*a*) Page 100: #5, 7, 8, 9, 10, 31.

(*b*) Try #17. See the discussion after Example 3. First you are trying to solve $A\mathbf{x} = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$.

(*c*) Try #23. Given $A\mathbf{x} = \mathbf{0}$. Multiply both sides of this equation on the LEFT by C. Now use the other piece of information in the problem.

(*d*) Suppose that A is $m \times n$ and that $CA = I_n$. What size is C? Now suppose that $AD = I_m$. What size is D? Compute CAD two ways: as $(CA)D$ and $C(AD)$. So what can you say about about the matrices C and D? What can you say about their sizes? Now what can you say about the size of A? Now do problem #25.

Hand In Wednesday

☞ Begin WeBWorK LHW6 online, due Thursday. Work on the problem set due Friday (see back of page). Come to the talk on Wednesday.

1. Assume B is a 3×3 matrix and that $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 5 & 12 \\ -1 & -2 & -3 \end{bmatrix}$. Suppose that $AB = I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 \end{bmatrix}$. Determine the three columns of B as follows.

(*a*) Solve the system of equations $A\mathbf{x} = \mathbf{e}_1$, where, as usual, $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

(*b*) Now solve the systems $A\mathbf{y} = \mathbf{e}_2$ and $A\mathbf{z} = \mathbf{e}_3$.

(*c*) Using the vectors you found above, form the matrix $B = [\mathbf{x}\ \mathbf{y}\ \mathbf{z}]$. Then calculate AB and BA.

(*d*) What's interesting about your answers above? How would you describe the relationship between A and B?

(*e*) Use a theorem in Section 2.1 to quickly evaluate A^TB^T and B^TA^T without and further numerical calculations.