Math 204: Day 17: Reading and Practice

Office Hour Help: M & W 2:30–4:00, Tu 2:00–3:30, & F 1:30–2:30 or by appointment. Website: http://math.hws.edu/~mitchell/Math204S16/index.php.

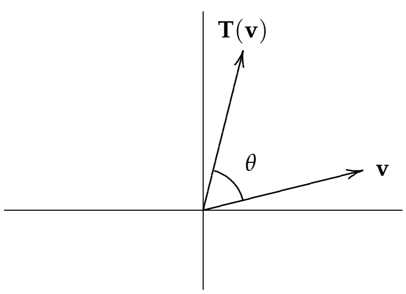
1. (*a*) Key terms and concepts today: If it acts like the inverse, it is the inverse! Elementary matrix. Determinant and inverses of 2×2 matrices. Invertibility implies Uniqueness/Existence of Solutions. “Socks and Shoes” Theorem.

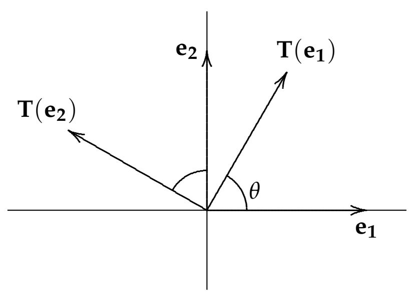
(*b*) In Section 2.2 try page 109 #3, 6, 9, 11 (X is a matrix. This is good practice in proving uniqueness, first prove that it is a solution and then assume two solutions exist and show they must be the same. . . as we did with inverses. See the proof of Theorem 5.), 13, 15 (Generalize “Socks and Shoes”), 17 (Be careful!), 21 (Use Theorem 2.5).

2. Read Section 2.3. It is essentially one BIG theorem that combines most of what we have done this term. After reading the section try page 115ff #15–32. These are good problems.

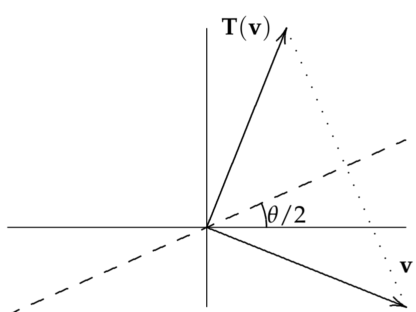
Extra Credit

1. An **orthogonal matrix** is a square matrix whose transpose is equal to its inverse: $Q^T = Q^{-1}$. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the transformation that rotates the plane θ radians about the origin. Let A be the standard matrix for T. Prove that A is orthogonal. Hint: Remember $A = [T(\mathbf{e}_1)\ T(\mathbf{e}_2)]$, so you should be able to figure out the entries of A in terms of trig functions and the angle θ.





2. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the transformation that reflects the plane in the line through the origin at angle $\theta/2$ radians. Let A be the standard matrix for T. Prove that A is orthogonal (see the previous problem). You will need to use some geometry and trig identities.



OVER

Assignment 10. Problems 1 to 5 due Monday. The others are due Wednesday. I will add a couple more on Monday.

1. Section 2.2, Exercises 2, 4, and 7. For 7(a) see Example 4 in the text. For 7(b) reduce the entire "super augmented" matrix $[A\ \mathbf{b}_1\ \mathbf{b}_2\ \mathbf{b}_3\ \mathbf{b}_4]$ all at once. Check that the solutions you found in part (a) appear in the last four columns.

2. Suppose that A is an $n \times n$ invertible matrix and c is a non-zero scalar. Show that cA is invertible by finding a formula (similar to those of Theorem 2.6) for $D = (cA)^{-1}$. Verify that your formula works; check that $D(cA) = I$ and $(cA)D = I$.

3. Assume that $T : \mathbb{R}^n \to \mathbb{R}^n$ is a linear transformation with standard matrix A. Prove: If A is invertible, then T is a one-to-one. Hint: One method is to use the One-to-One Dictionary to show that if A is invertible, then A satisfies one of the other conditions of equivalent to T being one-to-one. (Look at the theorems we proved Friday.)

Instructions. These next three problems use basic properties of matrix multiplication. You need to be aware that you are using such properties, because it is easy to make mistakes. Matrix multiplication is more complicated than ordinary scalar multiplication. With this in mind: Justify matrix any multiplication properties used by referring to an appropriate theorem in Section 2.1 or 2.2. E.g., "$A(BC) = (AB)C$ by associativity (Theorem 2.1(a))" or "$IC = C$ by Theorem 2.1 (e)" or "Since A is invertible, $(A^{-1})^{-1} = A$ by Theorem 2.6. Each time you multiply mention the side. E.g., "Left-multiply both sides of the equation by C." If you multiply by the inverse of a matrix, mention why you know the inverse exists. E.g., "Since D is the product of invertible matrices, by Theorem 2.6, D is invertible. So we can right-multiply by D^{-1}." Or "We are given that C is invertible, so C^{-1} exists." Here's an example:

EXAMPLE 0.0.1. Suppose that $(B - C)D = 0$, where B and C are $m \times n$ matrices and D is an $n \times n$ invertible matrix. Prove that $B = C$.

Proof. Given $(B - C)D = 0$, where B and C are $m \times n$ matrices and D is invertible matrix. (Show $B = C$.) Since D is invertible, D^{-1} exists so we can right-multiply by D^{-1} to get:

$$(B - C)(DD^{-1}) = 0D^{-1} = 0.$$

By definition of inverse, $DD^{-1} = I$ so we get $(B - C)I = 0$. Since I is the identity matrix, by Theorem 2.2 (e),

$$(B - C)I \overset{2.2(e)}{=} B - C = 0, \text{ so } B = C + 0 = C \text{ by Theorem 2.1 (c).}$$

□

4. Section 2.2, Exercise 8.

5. Section 2.2, Exercise 12. This proof should be short, but make sure you justify each step very precisely.

—————— Wednesday (More Problems to Be Added ——————

6. Section 2.2, Exercise 16. Note that you CANNOT say, "since AB is invertible, then $(AB)^{-1} = B^{-1}A^{-1}$," because you do not know that A^{-1} exists — you are trying to PROVE that A is invertible. (You can only apply the Socks-Shoes Theorem when you already know that the two matrices in question are both invertible.) After you use the Hint in the text, make sure you explain why the matrix you found is invertible.

7. Find the inverse of the matrix D in Exercise 11 of Section 2.1 and the inverse of the matrix in Section 2.2, Exercise 32. Show all work.