Math 204: Day 18 Recap and Agenda

Office Hour Help: M & W 2:30–4:00, Tu 2:00–3:30, & F 1:30–2:30 or by appointment. Website: `http://math.hws.edu/~mitchell/Math204S16/index.php`.

1. Key terms and concepts last time:

(*a*) First Connection (Theorem 2.5): A invertible $\Rightarrow$ for every $\mathbf{b} \in \mathbb{R}^n$, the system $A\mathbf{x} = \mathbf{b}$ has a *unique* solution.

(*b*) "Socks and Shoes" (Theorem 2.6): If $A_1, \ldots, A_p$ are all $n \times n$ invertible matrices, then $A_1 \cdots A_p$ is invertible and $(A_1 \cdots A_p)^{-1} = A_p^{-1} \cdots A_1^{-1}$. Note the order!

(*c*) Definition: E is an **elementary matrix** if it is obtained from I_n using a single row operation.

(*d*) Elementary Fact 1: If an elementary row operation is performed on A, the resulting matrix is the same as EA, where E is the elementary matrix by multiplying on the left by the corresponding elementary matrix E.

2. Key terms and concepts today:

(*a*) Elementary Fact 2: Elementary matrices are invertible.

(*b*) Second Connection (Theorem 2.7): A is invertible $\iff A \sim I_n$

(*c*) If A is invertible its inverse can be found using reduction: $[A\ I] \sim [I\ A^{-1}]$.

Reading, Practice, and Work

WeBWorK ShortElementaryMatrices. Due Thursday. Remainder of Assignment 10 (see back of sheet).

1. (*a*) Review Section 2.2. You should be comfortable finding inverses for any size matrix.

(*b*) In Section 2.2 try page 109 #3, 6, 9, 11 (X is a matrix. This is good practice in proving uniqueness, first prove that it is a solution and then assume two solutions exist and show they must be the same... as we did with inverses. See the proof of Theorem 5.), 13, 15 (Generalize "Socks and Shoes"), 17 (Be careful!), 21 (Use Theorem 2.5).

(*c*) Use the reduction algorithm to find the inverses in Section 2.2 110 #29, 31, and 33.

2. Read Section 2.3. It is essentially one BIG theorem that combines most of what we have done this term. After reading the section try page 115ff #15–32. These are good problems.

Remainder of Assignment 10 Due Wednesday

WeBWorK ShortElementaryMatrices. Due Thursday.

6. Section 2.2, Exercise 16. Note that you CANNOT say, "since AB is invertible, then $(AB)^{-1} = B^{-1}A^{-1}$," because you do not know that A^{-1} exists — you are trying to PROVE that A is invertible. (You can only apply the Socks-Shoes Theorem when you already know that the two matrices in question are both invertible.) After you use the Hint in the text, you will find that A equals some other matrix. Explain why this other matrix is invertible. So what can you conclude about A?

7. Find the inverse of the matrix D in Exercise 11 of Section 2.1 and the inverse of the matrix in Section 2.2, Exercise 32. Show all work.

8. ☞ These problems concern the determinant of 2×2 matrices. Use the formula after Theorem 4 in the text (p 103).

(*a*) Prove the following. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be an arbitrary 2×2 matrix and let r be a scalar. Then $\det(rA) = r^2 \det A$. Just write rA and apply the formula.

(*b*) Prove the following. Let A be an arbitrary 2×2 matrix. Then $\det A^T = \det A$.

(*c*) Bonus: Prove the following. Let A be an arbitrary 2×2 **invertible** matrix. Then $\det A^{-1} = \dfrac{1}{\det A}$.

9. (*a*) Which of the following matrices are elementary matrices?

(1) $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (3) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -3 \end{bmatrix}$ (4) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ (5) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

(*b*) For each of the elementary matrices above, determine its inverse using the idea at the top of page 107.

(*c*) The matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is the product of two elementary matrices E_2E_1.

Determine those two matrices and then use them to determine A^{-1}. Careful: Remember "Socks and shoes."