Office Hour Help: M & W 2:30–4:00, Tu 2:00–3:30, & F 1:30–2:30 or by appointment. Website: http://math.hws.edu/~mitchell/Math204S16/index.php.

Reading, Practice, and Work

Finish WeBWorK ShortElementaryMatrices (due Thursday). Begin Assignment 11 (see back of sheet and online for an Extra Credit problem).

1. Key concepts from last time:

(*a*) Elementary Fact 2: Elementary matrices are invertible.

(*b*) Second Connection (Theorem 2.7): A is invertible $\iff A \sim I_n$

(*c*) If A is invertible its inverse can be found using reduction: $[A\ I] \sim [I\ A^{-1}]$.

2. Key concepts today:

(*a*) Let A be $n \times n$. A is invertible if and only if A is a product of elementary matrices.

(*b*) The Connections Theorem (The Invertible Matrix Theorem, IMT, Theorem 2.8). A list of many conditions equivalent to A being invertible.

3. Re-read Section 2.3. It is essentially one BIG theorem that combines most of what we have done this term. After reading the section try page 115ff #15–32. These are good problems.

4. Skip to and read Section 3.1 on Determinants. Close reading exercise: Try Section 3.1, page 167 #1 and 3.

In Class: Group Work

These two problems are similar to problems on the next assignment.

1. Prove the following statements. Your proofs should be short but rigorous: Be sure you cite theorems to justify your claims. If you use the Connections Theorem, state explicitly which parts you are using as follows. Example: "Since A^T is invertible, by the Connections Theorem A the columns of A are independent."

(*a*) Let A be an $n \times n$ matrix. If A^T is a product of elementary matrices $E_p \cdots E_1$, then the columns of A span $\mathbb{R}^n$.

(*b*) Let A and B be $n \times n$ matrices. If B has pivots in every row and $A\mathbf{x} = 0$ has only the trivial solution, then BA is invertible.

(*c*) Let A be an $n \times n$ matrix. If A^T is not row equivalent to I, then the ROWS of A do not span $\mathbb{R}^n$.

2. One of these things is not like the others: In two of the following, it is impossible to give an example that meets the stated criteria, while in the third, an example is possible. Give an example in the one case where it is possible to do so, and PROVE that your example fits the bill. For the remaining two cases, PROVE that it is impossible to give an example. Your proofs should be short but rigorous. Make sure you cite theorems to justify your claims; if you use the Connections Theorem, state explicitly which parts you are using in the style described in the previous problem.

(*a*) A 4×4 matrix A with 3 pivots that is a product of elementary matrices.

(*b*) A 4×4 nonsingular matrix A whose inverse is a product of elementary matrices.

(*c*) A 4×4 matrix A with two identical ROWS that is invertible.

Assignment 11, Part One.

1. (Review problem.) Consider the matrix $C = \begin{bmatrix} 2 & 1 & -3 & 5 \\ 0 & 0 & 7 & 1 \\ 0 & 0 & 0 & 4 \end{bmatrix}$. Assume that the matrix C is the standard matrix for transformation T from $\mathbb{R}^n$ to $\mathbb{R}^m$.

(*a*) What are the values of n and m?

(*b*) Is T one-to-one? Justify your answer very clearly, citing specific theorems.

(*c*) Is Tonto $\mathbb{R}^m$? Justify your answer very clearly, citing specific theorems.

2. Prove each of the following statements. Be sure you cite theorems to justify your claims. If you use the Connections Theorem, state explicitly which parts you are using as follows. Example: "Since A^T is invertible, by the Connections Theorem A the columns of A are independent."

Harsh grading if there is no justification!

(*a*) If A is an $n \times n$ matrix and $\mathbf{c} \in \mathbb{R}^n$ is a vector so that $A\mathbf{x} = \mathbf{c}$ is INconsistent, then A's columns are linearly dependent.

(*b*) If A is an $n \times n$ matrix and $\mathbf{c} \in \mathbb{R}^n$ such that $A\mathbf{x} = \mathbf{c}$ has more than one solution, then A's columns do not span $\mathbb{R}^n$.

(*c*) If A and B are $n \times n$ matrices such that each of $A\mathbf{x} = \mathbf{0}$ and $B\mathbf{x} = \mathbf{0}$ has only the trivial solution, then $AB\mathbf{x} = \mathbf{0}$ also has only the trivial solution.

(*d*) If A can be written as a product of elementary matrices, then A's columns are linearly independent.

(*e*) If $n \times n$ matrix A is invertible, then the ROWS of A span $\mathbb{R}^n$.

3. One of these things is not like the others: In three of the following, it is impossible to give an example that meets the stated criteria, while in the fourth, an example is possible. Give an example in the one case where it is possible to do so, and PROVE that your example fits the bill. For the remaining three cases, PROVE that it is impossible to give an example. Your proofs should be short but rigorous. Make sure you cite theorems to justify your claims; if you use the Connections Theorem, state explicitly which parts you are using in the style described in the previous problem.

(*a*) A linear transformation $T : \mathbb{R}^5 \to \mathbb{R}^5$ that is one-to-one but not onto $\mathbb{R}^5$.

(*b*) A 5×5 non-singular matrix A whose transpose is the product of elementary matrices.

(*c*) A 5×5 matrix A with 5 pivots such that A's first column is a linear combination of A's second and third columns.

(*d*) A onto linear transformation $T : \mathbb{R}^5 \to \mathbb{R}^5$ whose standard matrix has an all-zero column.

4. (This is not the same problem as on part 2 of Assignment 10.) Let A and B be $n \times n$ matrices. Prove: If AB is invertible, then so is A. (Note: You cannot use the Socks-Shoes Theorem and write $(AB)^{-1} = B^{-1}A^{-1}$, because the Socks-Shoes Theorem only applies when BOTH A and B are known to be invertible.) Big Hint: Here's one method. Use the definition of invertible from class on Wednesday February 24 (or see page 103) to obtain a useful matrix C. Then multiply AB by C (which side?) and use associativity and then the Connections Theorem to get the result.

5. Find the inverse of $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & \frac{1}{2} & 1 \\ 1 & 2 & 1 \end{bmatrix}$.