Office Hour Help: M & W 2:30–4:00, Tu 2:00–3:30, & F 1:30–2:30 or by appointment. Website: `http://math.hws.edu/~mitchell/Math204S16/index.php`.

Reading, Practice, and Work

Finish WeBWorK Determinants1 (due Monday) and WeBWorK Determinants2 Due Thursday. **See back for Homework.**

1. Key concepts from last time:

DEFINITION 0.0.1. The **determinant** of a 1×1 matrix $A = [a]$ is $\det A = a$. For $n \geq 2$, the **determinant** of the $n \times n$ matrix $A = [a_{ij}]$ is the alternating sum

$$\det A = |A| = \sum_{j=1}^{n} (-1)^{1+j} a_{1j} \det A_{1j} = a_{11} \det A_{11} - a_{12} \det A_{12} + \cdots (-1)^{1+n} a_{1n} \det A_{1n}.$$

$C_{ij} = (-1)^{i+j} a_{ij} A_{ij}$ is the **(i,j)-cofactor** of A, then

$$\det A = |A| = \sum_{j=1}^{n} a_{1j} C_{1j} = a_{11} C_{11} + a_{12} C_{12} + \cdots + a_{1n} C_{1n}.$$

THEOREM 0.0.2 (Cofactor Expansion)**.** If A is an $n \times n$ triangular matrix, then $\det A = a_{11} \cdot a_{22} \cdots a_{nn}$, that is, $\det A$ is the product of the diagonal entries of A.

THEOREM 0.0.3 (Triangular Determinants)**.** The determinant of an $n \times n$ matrix A can be computed by cofactor expansion along any row or any column.

2. Key concepts today:

THEOREM 0.0.4 (Row Ops and Determinants)**.** Let A is an $n \times n$ matrix.

(*a*) If a multiple of one row of A is added to another row to produce a new matrix B, then $\det B = \det A$.

(*b*) If two rows of A are interchanged to produce B, then $\det B = -\det A$.

(*c*) If one row of A is multiplied by k to produce B, then $\det B = k \det A$.

Using the last two theorems above and reduction to the echelon form of an $n \times n$ matrix A, we get:

THEOREM 0.0.5 (Another Connection: Determinants and Inverses)**.** An $n \times n$ matrix A is invertible if and only if $\det A \neq 0$.

So we could add another condition to the Connections Theorem:

(*q*) $\det A \neq 0$

3. Read Section 3.2 and review Section 3.1 on Determinants.

(*a*) Practice: Page 177 Concepts: #1–4 Calculations: #5, 7, 11. Like an easy test question: #15, 17, 19. Read in the text if we don't get this far #21, 23, 25.

Class Work

1. Determine the following determinants using (*A*) reduction or (*B*) reduction and cofactor expansion combined. Check your answer with a partner.

$$A = \begin{vmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 7 & 6 & -3 \\ -3 & -10 & -7 & 2 \end{vmatrix} \qquad B = \begin{vmatrix} 1 & 5 & 4 & 1 \\ 0 & -2 & -4 & 0 \\ 3 & 5 & 4 & 1 \\ -6 & 5 & 5 & 0 \end{vmatrix}$$

2. Prove: If A is $n \times n$, then $\det A^T = \det A$.

Assignment 12

Due Wednesday in Class.

1. This question reviews several ideas. Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation. Let $\mathbf{u} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$. Assume that $T(\mathbf{u}) = T \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$ and $T(\mathbf{v}) = T \begin{bmatrix} 4 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

(*a*) Determine $T(3\mathbf{u} - 2\mathbf{v})$. Avoid using matrix multiplication.

(*b*) Determine $T(\mathbf{e}_1)$. Hint: Express $\mathbf{e}_1$ as a linear combination of $\mathbf{u}$ and $\mathbf{v}$. Then proceed as in part (a).

(*c*) Similarly determine $T(\mathbf{e}_2)$.

(*d*) Determine the standard matrix A for T.

(*e*) Is A invertible?

(*f*) Determine all the vectors $\mathbf{w}$ such that $T(\mathbf{w}) = \mathbf{b} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$.

(*g*) Determine $T(\mathbf{x}) = T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

(*h*) Is T onto? Explain carefully citing appropriate theorems.

(*i*) Is T one-to-one? Explain carefully citing appropriate theorems.

2. (*a*) Section 3.2 page 175, #14. Also decide whether the matrix is invertible.

(*b*) Section 3.2 page 175, #18, 20. Read the instructions. Use a theorem, not a calculation to determine the answers.

(*c*) Section 3.2 page 175, #22.

(*d*) Section 3.2 page 175, #26.