Office Hour Help: M & W 2:30–4:00, Tu 2:00–3:30, & F 1:30–2:30 or by appointment. Website: `http://math.hws.edu/~mitchell/Math204S16/index.php`.

Reading, Practice, and Work

1. Read Section 4.1 on Vector Spaces and Subspaces. This material is more abstract.

 (*a*) Close reading exercises: Try Practice Problem #1 on page 195 and then try Exercise #1 on the same page.

Assignment 13

Due Wednesday after break. Prove the following theorems.

1. Prove: If A is $n \times n$, then $\det A^T = \det A$. (This means we can do column operations on a matrix to simplify calculating the determinant.)

2. Prove: If A is $n \times n$ and invertible, then $\det A^{-1} = \dfrac{1}{\det A} = (\det A)^{-1}$.

3. Prove: If A is square matrix n is a positive integer, then $\det A^n = (\det A)^n$. If you know how to do induction, try it that way.

4. Let A is $n \times n$ and let k be any scalar. Find a formula for $\det(kA)$ in terms of $\det A$.

5. A $n \times n$ invertible matrix U is **orthogonal** if $U^{-1} = U^T$. Prove that if U is orthogonal, then $\det U = \pm 1$.

6. An $n \times n$ matrix A is said to be **conjugate** to the $n \times n$ matrix B if there is some invertible $n \times n$ matrix M so that $A = MBM^{-1}$. Prove: If A is conjugate to B, then $\det A = \det B$.

7. Page 176, #40.

8. **Sebastien's Theorem.** Consider these three **off-diagonal** matrices below.

$$A = \begin{bmatrix} 0 & 0 & 0 & a \\ 0 & 0 & b & 0 \\ 0 & c & 0 & 0 \\ d & 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 & 0 & 0 & a \\ 0 & 0 & 0 & b & 0 \\ 0 & 0 & c & 0 & 0 \\ 0 & d & 0 & 0 & 0 \\ f & 0 & 0 & 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & a \\ 0 & 0 & 0 & 0 & b & 0 \\ 0 & 0 & 0 & c & 0 & 0 \\ 0 & 0 & d & 0 & 0 & 0 \\ 0 & f & 0 & 0 & 0 & 0 \\ g & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

In each case, is the determinant of each the product of the diagonal elements? Show your work, justify your answer. Hint: Use row operations.

9. Bonus: Can you figure out for which size matrices Sebastien's Theorem holds?