Office Hour Help: M & W 2:30–4:00, Tu 2:00–3:30, & F 1:30–2:30 or by appointment. Website: http://math.hws.edu/~mitchell/Math204S16/index.php.

Reading and Practice

1. (*a*) Finish reading Section 4.3.

(*b*) Key definitions so far in Chapter 4: Vector space, subspace, null space of matrix A, and column space of matrix A, linear transformation between two general vector spaces, kernel and range of a linear transformation. The key term in the next section is **basis** for a vector space. Study that definition on page 209.

(*c*) Key theorems so far in Chapter 4: (1) Span$\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ is a subspace of $\mathbb{V}$; (2) If A is $m \times n$, then Nul A is a subspace of $\mathbb{R}^n$. (3) If A is $m \times n$, then Col A is a subspace of $\mathbb{R}^m$. (4) The kernel and range of a linear transformation $T : V \to W$ are subspaces of V and W, respectively.

In Class Example

Continuation from Friday: $\mathbb{P}_2 = \{\mathbf{p}(t) = a + bt + ct^2 : a, b, c \in \mathbb{R}\}$ is just the set of polynomials of degree 2 or less. Define $T : \mathbb{P}_2 \to \mathbb{R}^2$ by $T(\mathbf{p}) = \begin{bmatrix} \mathbf{p}(0) \\ \mathbf{p}(-1) \end{bmatrix}$.

(*a*) Find three vectors (polynomials) that span $\mathbb{P}_2$.

(*b*) If $\mathbf{q}(t) = 2 + 3t + 6t^2$, what is $T(\mathbf{q})$?

(*c*) If $\mathbf{q}(t) = a + bt + ct^2$, what is $T(\mathbf{q})$?

(*d*) Show that T is a linear transformation.

(*e*) Determine $\ker T$. Can you find a spanning set for $\ker T$.

(*f*) What is the Range of T? Is T onto?

Hand in Friday

1. Let $\mathbb{W} = \left\{ \begin{bmatrix} a+2b \\ a-b+2c \\ b+c \\ c-a \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$. Determine whether $\mathbb{W}$ is a subspace of $\mathbb{R}^4$. Show some work. Which method is easy?

2. (*a*) Let $T : M_{2\times 2} \to M_{2\times 2}$ by $T(A) = A + A^T$. Is T a linear transformation?

(*b*) Find a description in terms of the components for the matrices are in $\ker T$. That is determine

$$\ker T = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : \text{some condition on } a, b, c, d \right\}$$

Bonus if you can write $\ker T$ as the span of a set of vectors (matrices) in $M_{2\times 2}$. (See the hint for Problem 8.)

3. (*a*) Page 206 #6.

(*b*) Find a basis for Nul A. Read page 211.

4. Page 206 #26 (a to e). Each question is answered explicitly in the text. Quote the appropriate sentence in the text and give its page number. One of these is a bit sneaky.

5. Define $T : \mathbb{P}_2 \to \mathbb{R}^2$ by $T(\mathbf{p}) = \begin{bmatrix} \mathbf{p}(0) \\ \mathbf{p}(2) \end{bmatrix}$. (You may assume that T is a linear transformation.)

(*a*) If $\mathbf{p}(t) = 2 + 3t + 6t^2$, what is $T(\mathbf{p})$?

(*b*) If $\mathbf{q}(t) = a + bt + ct^2$, what is $T(\mathbf{q})$?

(*c*) Find a polynomial in $\mathbb{P}_2$ that spans $\ker T$. Show your work.

6. This is abstract but easy. Page 207 #30. Hint: Remember

$$\text{Range } T = \{\mathbf{u} \in \mathbb{W} : \mathbf{u} = T(\mathbf{v}) \text{ for some } \mathbf{v} \in \mathbb{V}\}.$$

So let's verify the three subspace properties:

(*a*) Why do we know that $\mathbf{0}_W$ is in Range T? (What vector $\mathbf{v}$ in $\mathbb{V}$ do we know for a FACT must get mapped to $\mathbf{0}_W$ by T?)

(*b*) Closure under addition: Let $\mathbf{w}, \mathbf{z} \in$ Range T. Show $\mathbf{w} + \mathbf{z} \in$ Range T. (If $\mathbf{w}, \mathbf{z} \in$ Range T, what does this mean? Use the definition above. So what element in $\mathbb{V}$ gets mapped to $\mathbf{w} + \mathbf{z}$ by T?)

(*c*) Closure under scalar multiplication. Let $\mathbf{w} \in$ Range T. What element in $\mathbb{V}$ gets mapped to $c\mathbf{w}$?

7. These are straightforward to check your understanding of bases for Nul A and Col A and related spaces. Page 213–214 #4, 6, 8, 10, 14 (all the hard work is done), and 16 (Hint: Convert to finding a basis for Col A.)

8. Find a basis for the subspace of $M_{2\times 2}$ spanned by the matrices

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 3 & -1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 5 & -3 \\ 3 & -4 \end{bmatrix}, \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}.$$

Gigantic Hint: Since we add and scalar multiply matrices just like we do with vectors in $\mathbb{R}^n$, we can *think of matrices in $M_{2\times 2}$ as vectors in* $\mathbb{R}^4$ by listing all of the matrix entries *vertically* in a column. Now you can solve the problem in $\mathbb{R}^4$ and convert the answer back to matrices.

9. **Extra Credit**: Let $T : \mathbb{R} \to \mathbb{R}^+$ by $T(x) = e^x$. Show that T is a linear transformation. Important: Remember $\mathbb{R}^+$ is the set of positive reals where $x \oplus y = xy$ and $c \odot x = x^c$.

(*a*) What is $\ker T$?

(*b*) What is Range T?

10. **Small XC, if you need practice with linear transformations**: Let $\mathbb{U}$, $\mathbb{V}$, and $\mathbb{W}$ be vector spaces. Assume that $T : \mathbb{U} \to \mathbb{V}$ and $S : \mathbb{V} \to \mathbb{W}$ are both linear transformations. Prove that the composition $G : \mathbb{U} \to \mathbb{W}$ by $G(\mathbf{u}) = S(T(\mathbf{u}))$ is a linear transformation.