Math 204: Day 32

Review Section 4.3 and read Section 4.4. Concentrate on 216–217 and middle of 219 through Example 6.

Hand In Wednesday

1. Page 214–215 #20, 22 (True-False, give a page reference to the text for each), 24 and 30.

2. Harder: Page 215 #32. This would make a good test problem and it is an important result. Use Part 3 of the Three Kernel Facts Theorem about $\ker T$ that we proved in class.

3. **a)** Page 215 #34. Hint: This is like #20 above.

b) In $\mathbb{P}_3$, let $\mathbf{p}_1 = 1 + 0t + 0t^2 + t^3$, $\mathbf{p}_2 = -2 + t - t^2 - t^3$, $\mathbf{p}_3 = 6 - t + 2t^2 + 5t^3$, $\mathbf{p}_4 = 5 - 3t + 3t^2 + 2t^3$, and $\mathbf{p}_5 = 0 + 3t - t^2 + 3t^3$. Find a basis for the space spanned by the given vectors. (Hint: Convert to finding a basis for Col A. Be sure to convert your answer back to polynomials.)

c) (Harder Think about the Spanning Set Theorem 5): In the vector space of all continuous functions on $(-\infty, \infty)$, find a basis for the subspace $\mathbb{H}$ spanned by $\{\sin^2 t, \cos^2 t, 1, \cos(2t)\}$. You will need trig identities from a calculus book or on line.

4. Find a basis for Nul A and Col A, where

$$A = [\mathbf{a}_1 \ldots \mathbf{a}_5] = \begin{bmatrix} 1 & 2 & 1 & 1 & 1 \\ 3 & 6 & 4 & 3 & 1 \\ 4 & 8 & 5 & 5 & 5 \\ 7 & 12 & 7 & 7 & 7 \end{bmatrix}.$$

5. Page 222 #4 (read the problem carefully) and 8 (different than #4).

6. Most of these are straightforward. First read the bottom of page 218 and top of 219

a) Carefully reread the bottom of page 218 and top of 219. Now try page 223 #10, 12, 14, and 32 (remember to convert your final answer to a polynomial).

b) Carefully reread the bottom of page 218 and top of 219. Now do page 223 problems #21 and 22. What is the matrix A in each case?

7. **Maple Extra Credit**. These both could be done by hand, but I want you do do them with Maple (or other software). In Maple, remember use fractions (not decimals).Page 224 #37 and 38.

8. Extra Credit: From Section 4.5. Read ahead and do any of page 229 #6, 12, 14, 22, and 24.

In Class

1. **a)** Find a basis for the subspace of $\mathbb{R}^4$: $\text{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 2 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \end{bmatrix} \right\}$

b) Find a basis for the subspace of $M_{2\times 2}$: $\text{Span}\left\{ \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \right\}$

c) Find a basis for the subspace of P_3: $\text{Span}\{1 + t + t^2, 2 + t + t^2 + t^3, 2 + 2t + 2t^2, 1 + t + t^2 + t^3, 1 + 2t^3\}$

d) Can you generalize what is common to all these examples? Think about this as you read Section 4.4.

Theorem 8

Let $\mathcal{B} = \{\mathbf{b}_1, \ldots, \mathbf{b}_n\}$ be a basis for $\mathbb{V}$. Then the coordinate mapping $T : \mathbb{V} \to \mathbb{R}^n$ by $T(\mathbf{x}) = [\mathbf{x}]_{\mathcal{B}}$ is a linear transformation that is one-to-one and onto, i.e., T is an isomorphism.

Proof (Linear): To show that T is linear check the two properties. Take any two vectors $\mathbf{u}, \mathbf{v} \in \mathbb{V}$. Since $\mathcal{B}$ is a basis, we can write

$\mathbf{u} =$ ________________________________, $\mathbf{v} =$ ________________________________.

So

$\mathbf{u} + \mathbf{v} =$ ____.

Then

$$T(\mathbf{u}+\mathbf{v}) = [\mathbf{u}+\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} + \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} = \underline{\qquad\qquad} = \underline{\qquad\qquad}.$$

Next $r\mathbf{u} =$ ________________________________ $=$ ________________________________. So

$$T(r\mathbf{u}) = [r\mathbf{u}]_{\mathcal{B}} = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} = \quad \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} = \underline{\qquad\qquad} = \underline{\qquad\qquad}.$$

Proof (One-to-one): To show that T is one-to-one we can use Fact 2: T is one-to-one if and only if $\ker T =$ ____________. So let $\mathbf{u} \in \ker T$. Show that $\mathbf{u} =$ ______. Since $\mathcal{B}$ is a basis, we can write

$\mathbf{u} =$ ________________________________.

Since $\mathbf{u} \in \ker T$,

$$\begin{aligned} T(\mathbf{u}) &= \underline{\qquad\qquad} \\ \\ &= \\ \\ &= \end{aligned}$$

So each $c_i = 0$ and so $\mathbf{u} = 0\mathbf{b}_1 + \cdots + 0\mathbf{b}_n$, so $\mathbf{u} =$ __________.

Proof (Onto): To show that T is onto, we take $\mathbf{x} \in \mathbb{R}^n$. Find $\mathbf{u} \in \mathbb{V}$ so that ____________. This is the same as saying, find $\mathbf{u}$ so that

$$[\mathbf{u}]_{\mathcal{B}} = \mathbf{x} = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}.$$

Which $\mathbf{u}$ will work? ______________________________________. Check $T(\mathbf{u}) = [\mathbf{u}]_{\mathcal{B}} = \qquad =$