Math 204: Day 42

Office Hours. Tuesday, Wednesday, and Thursday: 11:30–2:00.

1. (*a*) Review Sections 5.2 and 5.3. Review the Case Study on Owl populations (handout). Skim section 5.6 pages 301–middle of 303, 307(Owls)–309 in your text.

2. (*a*) **Practice**. Page 286ff: #1(see Example 2), 3, 5, 7, 9, 11, 17, 21.

(*b*) Previously suggested: **Practice**. First try Page 273 #1, 3, 7, 9, 15. Now go back and try Page 271 #3, 7, 9, 13, 17. Now try Page 271 21, 23 (for 2×2 only). Now try Finally try #35. This shows the utility of a basis of eigenvectors.

(*c*) I have put a number of **practice problems online** along with the answers. They do not cover all the material, but you may find them useful. There are also some additional problems from around the time of Exam 3.

3. The Final Exam is cumulative. About 30% will be on the most recent material. You should find that much of the earlier material is incorporated into more recent work.

(*a*) Key definitions to know from the past: row equivalent, RREF, linear combination, span, linear independence, basis, linear transformation, elementary matrix, transpose, finite-dimensional vector space, dimension, invertible matrix, rank, kernel, range, one-to-one, onto, isomorphism, stochastic and regular matrices. (You should also be familiar with the definition of a vector space.)

(*b*) More recently: eigenvalue, eigenvector, characteristic equation (polynomial), geometric multiplicity, similarity, diagonalizable matrix, probability vector, stochastic matrix.

4. Calculations and processes: You should prepared to do any of the following. Solving homogeneous and non-homogeneous systems $A\mathbf{x} = \mathbf{0}$ and $A\mathbf{x} = \mathbf{b}$; find the RREF of A; find the pivots of A; find A^{-1} if it exists; find $\det A$; find a basis for and dimension of Col A, Nul A, and Row A; find the eigenvalues and eigenvectors of A, diagonalize A (if possible); determine whether $T : \mathbb{V} \to \mathbb{W}$ is a linear transformation; determine whether $\mathbb{W}$ is a subspace of $\mathbb{V}$; determine $[\mathbf{x}]_B$; determining the kernel and range of a linear transformation; find the standard matrix for the linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$.

5. Random thoughts: pivots and free variables; row operations; cofactors; basic matrix algebra properties; transpose properties, determinant properties, matrix multiplication is not commutative;

6. You can find a (relatively) complete list of definitions and theorems online. In the meantime, here are the major theorems: Connections Theorem (IMT greatly extended); Theorem 1.2; Theorem 1.4; Theorem 1.7; Theorem 1.8; Theorem 1.10, Theorem 1.11–12, Theorem 2.3; Theorem 2.4; Theorem 2.6; Theorem 2.7; Theorem 2.9; Theorem 3.3; Theorems 3.5–6; Theorem 4.1; Theorem 4.2; Theorem 4.3; Theorem 4.5; Theorem 4.6; Theorem 4.7; Theorem 4.8; Theorems 4.9–10; Theorems 4.11–12; Theorem 4.12; Theorem 4.13; Theorem 5.4; Theorem 5.5; Theorem 5.6., Theorem 5.7(b).

THEOREM 5.3.2 (Eigenvectors corresponding to Distinct Eigenvalues (Jack's Proof)). If $\mathbf{v}_1, \ldots, \mathbf{v}_p$ are eigenvectors corresponding to distinct eigenvalues $\lambda_1, \ldots, \lambda_p$ of A, then $\mathbf{v}_1, \ldots, \mathbf{v}_p$ are ______________________________.

THEOREM 5.3.5 (Diagonalization Theorem). An $n \times n$ matrix A is diagonalizable if and only if A has ______________________________ eigenvectors. In this situation $A = PDP^{-1}$ where

$$P = \begin{bmatrix} \quad & \quad & \quad \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} \quad & & \\ & \quad & \\ & & \quad \end{bmatrix}.$$

Combining the last two theorems:

THEOREM 5.3.6. If an $n \times n$ matrix A has n distinct eigenvalues, then A is ______________________________.

All of the stage-matrix examples exhibited this behavior.

THEOREM 5.3.7 ((b): Sum of the dimensions of the eigenspaces). An $n \times n$ matrix A is diagonalizable if and only if the sum of the dimensions of its eigenspaces is ____.

In practice, this is the theorem that you have been using to diagonalize (or not) matrices.

Markov Example. Suppose that 3% of the population of the U.S. lives in the State of Washington. Suppose the migration of the population into and out of Washington State will be constant for many years according to the following migration probabilities: 90% of those living in the state remain in the state each year. 1% of the population living in the rest of the U.S. moves to Washington state each year. What percentage of the total U.S. population will eventually live in Washington?

(*a*) What are we looking for?

(*b*) First, what is the initial state of the population? Does it matter?

(*c*) What is the Migration (Markov) matrix? Is it regular?

(*d*) Use your brain to find the steady state.