

# Math 204 Notes on Grammar

Subspace proofs: Suppose  $V$  is a vector space and

$$W = \{ \vec{u} \in V : \vec{u} \text{ satisfies condition A} \}$$

To prove  $W$  is a subspace of  $V$ ...

- a) Show that  $\vec{0}_V$  satisfies condition A (or not)
- b) Let  $\vec{u}, \vec{v} \in W$ . (1) So  $\vec{u}$  satisfies condition A and (2)  $\vec{v}$  satisfies condition A  
Now show  $\vec{u} + \vec{v}$  satisfies condition A (or not) by using (1) and (2)
- c) Let  $\vec{u} \in W$ . (1) So  $\vec{u}$  satisfies condition A. Let  $c$  be any scalar. Use (1) to show  $c\vec{u}$  satisfies condition A (or not)

Transformation proofs:

When trying to verify that  $T(\vec{u} + \vec{v}) = T(\vec{u}) + T(\vec{v})$   
do NOT start by simply saying

$$"T(\vec{u} + \vec{v}) = T(\vec{u}) + T(\vec{v})"$$

You don't know this is true (yet). Instead write  
"Let  $\vec{u}, \vec{v} \in V$   
Show  $T(\vec{u} + \vec{v}) = T(\vec{u}) + T(\vec{v})$ "

Foolproof Method: compute  $T(\vec{u} + \vec{v})$  and  $T(\vec{u}) + T(\vec{v})$   
separately and compare the results

Similarly: for scalar multiplication, start with  
"Let  $u \in V$  and  $c$  be a scalar  
Show  $T(c\vec{u}) = cT(\vec{u})$ "

Again compute each separately and compare will always work