$with(LinearAlgebra):$

The transition matrix is $P := \begin{bmatrix} \frac{1}{2} & \frac{1}{3} & 0 \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{2} \\ 0 & \frac{1}{3} & \frac{1}{2} \end{bmatrix}$:

Note the use of fractions to get exact values. While the mouse cannot go directly from cell 1 to 3 (or vice versa) it can get there in two moves, so we expect P^2 to have all positive entries.

P^2

$$\begin{bmatrix} \frac{5}{12} & \frac{5}{18} & \frac{1}{6} \\ \frac{5}{12} & \frac{4}{9} & \frac{5}{12} \\ \frac{1}{6} & \frac{5}{18} & \frac{5}{12} \end{bmatrix} \quad \textbf{(1)}$$

So P is regular. So it should have a unique steady-state vector.

$ReducedRowEchelonForm(P - IdentityMatrix(3))$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -\frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix} \quad \textbf{(2)}$$

The solution is $w := \begin{bmatrix} 1 \\ \frac{3}{2} \\ 1 \end{bmatrix}$: To eliminate fractions use $x_3 = 2$ so $w := \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}$:

So the steady state probability vector is $q := \frac{1}{7} w$

$$\begin{bmatrix} \frac{2}{7} \\ \frac{3}{7} \\ \frac{2}{7} \end{bmatrix} \quad \textbf{(3)}$$

Check: $P.q$

$$\begin{bmatrix} \frac{2}{7} \\ \frac{3}{7} \\ \frac{2}{7} \end{bmatrix} \quad \textbf{(4)}$$

Does P = [q q . . . q]? Here we use evalf to get a decimal answer: $evalf(P^{100})$

$$\begin{bmatrix} 0.2857142857 & 0.2857142857 & 0.2857142857 \\ 0.4285714286 & 0.4285714286 & 0.4285714286 \\ 0.2857142857 & 0.2857142857 & 0.2857142857 \end{bmatrix} \quad \textbf{(5)}$$

Yes, the colum entries are approximately 2/7, 4/7, 2/7.