

## Maple Primer 2: Other Commands to Solve Linear Systems

with(LinearAlgebra) :

### Review Maple Primer 1 if needed.

Once you have mastered how to perform row operations both in Maple and by hand, there are other ways to solve systems of equations in Maple without having to use the *RowOperation* function. Let A be the augmented matrix of the linear system from Example 1 on page 5 in the text.

$$A := \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ -4 & 5 & 9 & -9 \end{bmatrix} \qquad \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ -4 & 5 & 9 & -9 \end{bmatrix} \qquad (1)$$

We can find the reduced row echelon form of A (i.e., the solution of the system) in a single command:

**ReducedRowEchelonForm(A)**

*ReducedRowEchelonForm(A)*

$$\begin{bmatrix} 1 & 0 & 0 & 29 \\ 0 & 1 & 0 & 16 \\ 0 & 0 & 1 & 3 \end{bmatrix} \qquad (2)$$

Of course this means that  $x_1 = 29$ ,  $x_2 = 16$ , and  $x_3 = 3$ .

Now let's solve the system in Example 4 on page 18 whose augmented matrix (which is already in echelon form) is

$$M := \begin{bmatrix} 1 & 6 & 2 & 5 & -2 & -4 \\ 0 & 0 & 2 & -8 & -1 & 3 \\ 0 & 0 & 0 & 0 & 1 & 7 \end{bmatrix} \qquad \begin{bmatrix} 1 & 6 & 2 & 5 & -2 & -4 \\ 0 & 0 & 2 & -8 & -1 & 3 \\ 0 & 0 & 0 & 0 & 1 & 7 \end{bmatrix} \qquad (3)$$

*ReducedRowEchelonForm(M)*

$$\begin{bmatrix} 1 & 6 & 0 & 13 & 0 & 0 \\ 0 & 0 & 1 & -4 & 0 & 5 \\ 0 & 0 & 0 & 0 & 1 & 7 \end{bmatrix} \qquad (4)$$

This means that  $x_1 = -6x_2 - 13x_4$ ,  $x_2$  is free,  $x_3 = 5 + 4x_4$ ,  $x_4$  is free, and  $x_5 = 7$ .

Another way to solve a system is to enter the coefficient matrix and column vector of answers separately. Let's redo the previous example. Use the matrix palette to enter the 3 x 5 matrix of coefficients.

$$F := \begin{bmatrix} 1 & 6 & 2 & -5 & -2 \\ 0 & 0 & 2 & -8 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 6 & 2 & -5 & -2 \\ 0 & 0 & 2 & -8 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (5)$$

Then enter the 3 x 1 column vector using the matrix palette (make sure to choose 'Insert Vector [column vector]')

$$v := \begin{bmatrix} -4 \\ 3 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} -4 \\ 3 \\ 7 \end{bmatrix}$$

(6)

Now solve the system by using the command **LinearSolve(A,x)** with the appropriate inputs.  
 $LinearSolve(F, v)$

$$\begin{bmatrix} -6 - t_2 - 3 - t_4 \\ -t_2 \\ 5 + 4 - t_4 \\ -t_4 \\ 7 \end{bmatrix}$$

(7)

The **free variables** are indicated with t's and an underscore \_ The advantage of using *LinearSolve* is that if you have more than one system with the same coefficient matrix (but different solutions) then you can easily change the system. For example if  $w$  is the vector

$$w := \begin{bmatrix} -3 \\ 3 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} -3 \\ 3 \\ 7 \end{bmatrix}$$

(8)

Now we can solve the new system easily:  $LinearSolve(F, w)$

$$\begin{bmatrix} 1 - 6 - t_0_2 - 3 - t_0_4 \\ -t_0_2 \\ 5 + 4 - t_0_4 \\ -t_0_4 \\ 7 \end{bmatrix}$$

(9)

We knew  $x_2$  and  $x_4$  and would be free (why) and  $x_5 = 7$ , but now the other dependent variables have different solutions.

**Review: Using the *Matrix* command to construct a matrix.** Specify all element values as a list of lists that define the element values row-wise. Lists are enclosed in square brackets and commas are used to separate elements of the list. For example, to create a 2 x 3 matrix with 1 2 3 in the first row and 4 5 6 in the second row, we will need a list of two lists -- the two lists are the first row [1,2,3] and the second row [4,5,6] which are then put in a list, i.e. inside square brackets.

$$K := Matrix([[1, 2, 3], [4, 5, 6]])$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

(10)

Column vectors (denoted with lower case letters) are formed using angle brackets and commas (commas indicate new rows):

$z := \langle 7, 8 \rangle$

$$\begin{bmatrix} 7 \\ 8 \end{bmatrix} \quad (11)$$

Now we can solve the augmented system  $[K \mid z]$

$LinearSolve(K, z)$

$$\begin{bmatrix} -\frac{19}{3} + \_tI_3 \\ \frac{20}{3} - 2\_tI_3 \\ \_tI_3 \end{bmatrix} \quad (12)$$

### Building Augmented Matrices from Coefficient Matrices and Column Vectors

We can build an augmented matrix from the coefficient matrix  $A$  of a system and the column  $b$  of constants in the system in the following way. First create the matrix  $A$  as usual, let's say it is  $3 \times 3$ :

$$A := \begin{bmatrix} 27 & 99 & 92 \\ 8 & 29 & -31 \\ 69 & 44 & 67 \end{bmatrix}$$

$$\begin{bmatrix} 27 & 99 & 92 \\ 8 & 29 & -31 \\ 69 & 44 & 67 \end{bmatrix} \quad (13)$$

Now create the column vector  $b$  (a  $3 \times 1$  matrix) with either the matrix palette (as below) or angle brackets:

$$b := \begin{bmatrix} -32 \\ -74 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} -32 \\ -74 \\ -4 \end{bmatrix} \quad (14)$$

To create the augmented matrix, use angle brackets and a vertical bar (the vertical bar indicates a new column):

$Aug := \langle A \mid b \rangle$

$$\begin{bmatrix} 27 & 99 & 92 & -32 \\ 8 & 29 & -31 & -74 \\ 69 & 44 & 67 & -4 \end{bmatrix} \quad (15)$$

Now solve as usual:  $ReducedRowEchelonForm(Aug)$

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{54207}{163622} \\ 0 & 1 & 0 & -\frac{207597}{163622} \\ 0 & 0 & 1 & \frac{182389}{163622} \end{bmatrix} \quad (16)$$

This would be difficult by hand! Notice that we get an **exact** answer. **Don't use a decimal approximation** unless I ask for it. We could have solved this using

$LinearSolve(A, b)$

$$\begin{bmatrix} \frac{54207}{163622} \\ -\frac{207597}{163622} \\ \frac{182389}{163622} \end{bmatrix} \quad (17)$$

Why bother with the augmented method? Because if we now want to use the same coefficient matrix but a different set of constants in the column, we do not need to retype the entire matrix. Suppose we want to solve

$$C := \left\langle A \left| \begin{bmatrix} -76 \\ -72 \\ -2 \end{bmatrix} \right. \right\rangle$$

$$\begin{bmatrix} 27 & 99 & 92 & -76 \\ 8 & 29 & -31 & -72 \\ 69 & 44 & 67 & -2 \end{bmatrix} \quad (18)$$

Notice we did not first need to define the column vector; just slap A together with a column vector of our choice using the matrix palette. Now reduce:

`ReducedRowEchelonForm(C)`

$$\begin{bmatrix} 1 & 0 & 0 & \frac{26869}{163622} \\ 0 & 1 & 0 & -\frac{263481}{163622} \\ 0 & 0 & 1 & \frac{140477}{163622} \end{bmatrix} \quad (19)$$

We could even do this with a single command:

$$\text{ReducedRowEchelonForm} \left( \left\langle A \left| \begin{bmatrix} 57 \\ 27 \\ -93 \end{bmatrix} \right. \right\rangle \right)$$

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{652245}{327244} \\ 0 & 1 & 0 & \frac{421317}{327244} \\ 0 & 0 & 1 & -\frac{59205}{327244} \end{bmatrix} \quad (20)$$

or even

$$\text{LinearSolve} \left( A, \begin{bmatrix} 57 \\ 27 \\ -93 \end{bmatrix} \right)$$

$$\begin{bmatrix} \frac{652245}{327244} \\ -\frac{421317}{327244} \\ \frac{59205}{327244} \end{bmatrix} \quad (21)$$