Some Practice Problems

By no means are these comprehensive. These represent some leftover problems.

1. (*a*) Let $\mathbb{F}$ denote the set all functions that are defined for all real numbers. Let $\mathbb{W} = \{f \in \mathbb{F} \mid f(1) = 4f(2)\}$. Is $\mathbb{W}$ a subspace of $\mathbb{F}$?

(*b*) Let $\mathbb{W} = \{f \in \mathbb{F} \mid f(3) + f(2) = 1\}$. Is $\mathbb{W}$ a subspace of $\mathbb{F}$?

2. Let $\mathbb{P}$ be the set of all polynomials (of any degree). Define $T : \mathbb{P} \to \mathbb{P}$ by $T(\mathbf{p}) = t\mathbf{p}'(t)$. Determine whether T is a linear transformation.

3. Hint: In each of these questions, convert the given vectors into column vectors in $\mathbb{R}^n$ and solve the problem as you would in $\mathbb{R}^n$.

(*a*) Is $S = \{1 + x,\ x - x^2,\ 1 - x^2\}$ a basis for P_2? Justify your answer.

(*b*) Is $\left\{ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ a basis for $M_{2\times 2}$?

(*c*) Is $\left\{ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \right\}$ a basis for $M_{2\times 2}$?

(*d*) You should have found that S in part (a) was a basis. If $\mathbf{q} = 1 + 2x + 3x^2$, find $[\mathbf{q}]_S$

4. (*a*) Prove: If A and B are similar $n \times n$ matrices and A is not invertible, then B is not invertible.

(*b*) Suppose that A, B, and C are all $n \times n$ matrices. Prove: If A is similar to B and B is similar to C, then A is similar to C.

5. For what values of k, if any, is $B = \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ k \\ k \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ k \end{bmatrix} \right\}$ a basis for $\mathbb{R}^3$? Justify your answer. (This problem is similar to one on Exam 3, but it is not identical to it.

6. For each set of vectors, determine whether they are a basis for $\mathbb{R}^3$. If not, explain why.

(*a*) $\left\{ \begin{bmatrix} 3 \\ 1 \\ -4 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix} \right\}$

(*b*) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix} \right\}$

(*c*) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \right\}$

(*d*) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 3 \\ 8 \\ 9 \end{bmatrix} \right\}$

7. Explain why $\mathbb{W} = \left\{ \begin{bmatrix} a+b+c \\ 2a+3b-c \\ 2a+4b-4c \end{bmatrix} \in \mathbb{R}^3 : a, b, , c \in \mathbb{R} \right\}$ is a subspace of $\mathbb{R}^3$.

8. Markov chain: Page 260–261, #3 and 13, which go together.

9. Rank: Page 237, #7, 9, 15, and 19. (See answers in back of text.)

10. Diagonalize $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$ if possible by finding P and D.

11. Great Problem: Page 261, #17 (answer is in back of text).