Math 204: Practice Problems around the time of Exam 3

By no means is this complete! For example, there are no determinant property questions.

1. a) Find a basis for the subspace of H of $\mathbb{R}^3$ spanned by $\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 8 \\ 6 \\ 5 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 7 \end{bmatrix}$.

b) Find $\dim H$.

2. a) Let $B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \right\}$. Show that B is a basis for $\mathbb{R}^3$.

b) If $\mathbf{x} = \begin{bmatrix} 3 \\ -5 \\ 4 \end{bmatrix}$, determine $[\mathbf{x}]_B$.

3. Let $\vec{p}_1(t) = 1 + t^2$, $\vec{p}_2(t) = 2 - t + 3t^2$, $P_3(t) = 1 + t - 3t^2$.

a) Use coordinate vectors to show that these vectors are a basis for $\mathbb{P}_2$.

b) Consider the basis $\mathcal{B} = \{\vec{p}_1, \vec{p}_2, \vec{p}_3\}$ for $\mathbb{P}_2$. Find $\vec{q}$ in $\mathbb{P}_2$ given that $[q]_{\mathcal{B}} = \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}$.

4. If $A = \begin{bmatrix} -2 & 4 & -2 & -4 \\ 2 & -6 & -3 & 1 \\ -3 & 8 & 2 & -3 \end{bmatrix}$, find bases for Col A and Nul A and determine their respective dimensions.

5. Let $S = \left\{ \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}, \begin{bmatrix} 4 & 12 \\ 8 & 20 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}, \begin{bmatrix} -1 & 5 \\ 2 & 8 \end{bmatrix} \right\}$. Find a basis of Span S and find its dimension.

6. Page 223 #21.

7. Let $T : M_{2\times 2} \to \mathbb{R}^2$ by $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a + 2d \\ b + c - d \end{bmatrix}$. Find a basis for $\ker T$ and determine $\dim \ker T$.

8. a) Show that the transformation $T : \mathbb{R}^2 \to \mathbb{P}_2$ by $T\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = (a + b) + at + (b - a)t^2$ is linear.

b) Show that it is one-to-one. (Hint: One method is to determine $\ker T$.)

9. a) Determine whether $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T(\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = \begin{bmatrix} a + b \\ b^2 \end{bmatrix}$ is a linear transformation.

b) $T : \mathbb{P}_2 \to \mathbb{R}^2$ by $T(\mathbf{p}) = \begin{bmatrix} \mathbf{p}(1) \\ 2\mathbf{p}(0) \end{bmatrix}$ is a linear transformation. (XC Bonus: If so, what is $\ker T$? Hint: For this part use that $p(t) = a + bt + ct^2$. Do not use this for checking linearity.)

10. Is $\mathbb{W} = \left\{ \begin{bmatrix} a \\ a^2 \end{bmatrix} : a \in \mathbb{R} \right\}$ a subspace of $\mathbb{R}^2$?

11. Is $\mathbb{W} = \{\mathbf{p} \in \mathbb{P}_5 : \int_0^1 \mathbf{p}(t)\, dt = 1\}$ a subspace of $\mathbb{P}_5$?

12. Let $T : \mathbb{V} \to \mathbb{W}$ be a linear transformation. Prove that $\ker T$ is a subspace of $\mathbb{V}$. (We proved this in class, can you do the proof with looking it up?)

13. Page 196 #3. (The answer is correct, but it references the a page number from the earlier edition of the text.)

14. Return to practice problem #3 above. If $\mathbf{p}(t) = 3 + t + 5t^2$ determine $[\mathbf{p}]_B$.

15. (Think!) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation that is one-to-one. What is the dimension of the Range of T? (Hint: Think about the standard matrix for this transformation.)

16. (More thinking!) Suppose $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$, $\mathbf{p}_4$ are polynomials so that Span$\{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4\} = H$ is a two-dimensional subspace in $\mathbb{P}_5$. Describe how to find a basis for H by examining the four polynomials and doing almost no work.

17. Page 237, #21. (The answer is correct, but it again references the wrong page number.)