`Maple` Final Exam Problem Guidelines: You **must** work with your partner on these three problems. Work from groups of more or less than two will not be accepted. **You must not consult with anyone other than your partner and me. This is an exam.** Hand in one solution set for both of you. Make sure to **include explanatory text** using Text formatting in `Maple`. Use 10 point font size. Print on both sides if possible. Your work must be **stapled**. Your `Maple` output should be neat. Papers with extraneous or error-laden output will be penalized. If you need help, see me.

☞ The interface(displayprecision) command controls the number of decimal places to be displayed. Set it to display only 4 digits (though internally it is keeping track of more digits). Use

interface(displayprecision = 4)

Problems

1. **Case Study: Spotted Owl Update.l** The most recent spotted owl data available [Lamberson] yield the following entries for the stage matrix. Notice that a few subadults produce offspring (akin to teenage pregnancy in humans.) Using these data, determine whether the owl population is becoming extinct. If the population is not becoming extinct, determine the percentage of each class in the long-run population.

☞ Use fractions in your matrices.

8 points. Note: Point value subject to change depending on the total value of the final exam.

Juvenile Survival	.33
Subadult Survival	.85
Adult Survival	.85
Subadult Fecundity	.125
Adult Fecundity	.26

2. **Case Study: Blue Whale Survival.** In the 1930s (before its virtual extinction and a great change in its survival rates) a researcher studied the blue whale, *Balaenoptera musculus*, population and obtained the data below. Due to the long gestation period, mating habits, and migration of the blue whale, a female can produce a calf only once in a two-year period. Thus the age classes for the whale are assumed to be: less than 2 years, 2 or 3 years, 4 or 5 years, 6 or 7 years, 8 or 9 years, 10 or 11 years, and 12 or more years. The stage-matrix for the model is given below.

8 points

   (*a*) Determine whether the blue whale population is becoming extinct. If the population is not becoming extinct, determine the percentage of each class in the long-run population. [Caution: Look carefully at your `Maple` calculation. It may be helpful to widen your window or decrease the font size to see all the output.]

$$\begin{bmatrix} 0 & 0 & .19 & .44 & .50 & .50 & .45 \\ .77 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .77 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .77 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & .77 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & .77 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .77 & .78 \end{bmatrix}.$$

   (*b*) If the population is not becoming extinct, estimate what percent of the whale population could be sustainably harvested every year by indigenous populations while keeping the whale population constant.

3. **Case Study: Adelie Penguin Survival.** Now that you know about stage-matrices, you will see them in the popular press. The following information is derived from Fen Montaigne's article, "The Ice Retreat" that appeared in *The New Yorker* (Dec 21 & 28, 2009), pages 72–82 (see page 81). The article discusses the impact of shrinking sea ice on the Adelie penguin population along the Antarctic Peninsula. The (female) population is divided into three classes: chicks, yearlings, and adults. Survey data indicate that only 1 in 8 chicks survive. Yearlings survive at a rate 25% *lower* than the adult survival rate. The adult survival rate r is changing as the amount of sea ice changes. The fecundity rate for adult females is 0.65.

10 points

   (*a*) Create the stage-matrix for this model.

   (*b*) What adult survival rate r will produce a matrix with dominant eigenvalue $\lambda_1 = 1$? (If λ_1 dips below 1, the Adelie penguin population will be headed for extinction.)

   (*c*) Using this value of r, determine the long-run age-class distribution.

4. **Case Study: The Nightly News.** Darwin's idea of 'the survival of the fittest' was an underlying principle in his theory of natural selection in biology and the first two problems show how that plays out. Capitalists also argue that the market place is an arena where only the fittest (that is, those with a competitive advantage) survive. One very competitive arena is the quest for TV ratings or market share. Market shares are directly tied to the prices that networks can charge advertisers. The arena of "network news" is as competitive as any. The three major network anchors are all competing for viewers in exactly the same time slot. The following gives some idea of how networks might use Markov chains to predict future ratings. Suppose surveys indicate that: 12 points

- Of those who watch ABC one night, 70% are likely to watch ABC the next night, 30% switch to CBS, and 0% switch to NBC.
- Of those who watch CBS one night, 50% are likely to watch CBS the next night, 40% switch to ABC, and 10% switch to NBC.
- Of those who watch NBC one night, 60% are likely to watch NBC the next night, 30% switch to CBS, and 10% switch to ABC.

(*a*) Describe this situation in terms of a 3×3 transition matrix for a Markov Chain model.

(*b*) Determine whether the matrix is regular.

(*c*) Find the steady-state vector for the system.

(*d*) Interpret the steady-state vector.