Facts and Theorems for Test 1. Know and be able to use the following theorems.

See your text for the precise statements.

1. Uniqueness of Reduced Row-Echelon Form (Theorem 1.1)

2. Existence and Uniqueness Theorem (Theorem 1.2)

3. Properties of Scalar Multiplication and Vector Addition (p. 27)

4. (*a*) Equivalent Representations Theorem (Theorem 1.3) $A\mathbf{x} = \mathbf{b}$ is the same as a vector equation and is solved using the augmented matrix $[a\,\mathbf{b}]$. (p. 36)

(*b*) Corollary: $A\mathbf{x} = \mathbf{b}$ has a solution if and only if $\mathbf{b}$ is a linear combination of the columns of A (if and only if $\mathbf{b}$ is Span $\{\mathbf{a}_1, \ldots \mathbf{a}_n\}$.) (p. 36)

5. Spanning and Pivots Theorem: These are equivalent Theorem 1.4 (p. 37) :

(*a*) There are pivots in every ROW of A.

(*b*) Columns of A span $\mathbb{R}^m$

(*c*) $A\mathbf{x} = \mathbf{b}$ is consistent for EVERY $\mathbf{b} \in \mathbb{R}^m$.

(*d*) EVERY $\mathbf{b}$ in $\mathbb{R}^m$ is a linear combination of the columns of A.

6. Properties (Linearity) of Matrix-Vector Multiplication (Theorem 1.5)

7. Homogeneous Systems are Consistent (p. 43) and Homogeneous Solutions Theorem (p. 43): ($A\mathbf{x} = \mathbf{0}$ has a non-trivial solution if and only if there is a free variable.)

8. Independence of Matrix Columns (p. 57) iff $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

Remember: iff means if and only if.

9. Characterization of Dependent Sets. (p.58) (A vector in the set is a linear combination of the others.)

10. (*a*) Surplus of Vectors (More Vectors than Entries $\Rightarrow$ linear dependence) (p. 59)

(*b*) Dependence of Sets Containing the Zero Vector. (p. 59)

Definitions. Memorize the definitions of these terms for the exam: row equivalent matrices, linear independence (dependence), pivot position, the span of $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$, linear combination, homogeneous and nonhomogeneous system.

See the Glossary in the text.

Pivot Dictionary. Results about pivots:

(*a*) For a *particular vector* $\mathbf{b}$: No pivot in rightmost column of *augmented* $[A\ \mathbf{b}] \iff A\mathbf{x} = \mathbf{b}$ is consistent

(*b*) A has a pivot in every row $\iff$ $A\mathbf{x} = \mathbf{b}$ is ALWAYS consistent for all $\mathbf{b}$ $\iff$ the columns of A span $\mathbb{R}^m$

(*c*) The *coefficient matrix* A has a pivot in every column $\iff$ $A\mathbf{x} = \mathbf{0}$ has only the trivial solution $\iff$ the columns of A are independent