

Math 204: Test 1

Show all work on these pages to receive full credit. **Put your name on the last page only.**

1. a) (5 pts) Complete the definition: A transformation $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is **onto** if

b) (5 pts) Suppose that $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 + x_2 \\ 3x_2 \\ x_1 - x_2 \end{bmatrix}$ is a linear transformation. What is the standard matrix A for the transformation T ?

c) (5 pts) Using the transformation in part (b), determine the image of $\mathbf{x} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

d) (5 pts) Using the transformation in part (b), is $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$ in the range of T ?

e) (5 pts) Is the transformation in part (b) onto? Explain carefully.

2. a) (5 pts) Find the solutions to the system of equations below and express the answer in **parametric form**.

$$\begin{aligned}x_1 + 2x_2 + 2x_3 + x_4 &= 1 \\x_1 + 3x_2 + 3x_3 + 3x_4 &= 3 \\-2x_1 - 2x_2 - 2x_3 + 2x_4 &= 2\end{aligned}$$

- b) (5 pts) Without doing any additional work, give the solution in parametric form to the **homogeneous system**

$$\begin{aligned}x_1 + 2x_2 + 2x_3 + x_4 &= 0 \\x_1 + 3x_2 + 3x_3 + 3x_4 &= 0 \\-2x_1 - 2x_2 - 2x_3 + 2x_4 &= 0\end{aligned}$$

- c) (5 pts) Suppose that A is a 4×4 matrix and that there is a vector \mathbf{b} in \mathbb{R}^4 so that system of equations $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions. Prove that the columns of A do *not* span \mathbb{R}^4 .

3. a) (5 pts) Carefully complete the definition: The vector \mathbf{w} is in the **span** of $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_p\}$ if

b) (5 pts) Determine whether the vectors $\mathbf{v}_1 = \begin{bmatrix} -1 \\ 3 \\ 5 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 2 \\ 9 \\ 5 \end{bmatrix}$ span \mathbb{R}^3 .

c) (5 pts) Complete the definition: The set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_p\}$ is **linearly independent** if

d) (5 pts) Assume that A is a 5×5 matrix whose columns span \mathbb{R}^5 . Prove that the columns of A are linearly independent.

e) (5 pts) Determine all values of h for which the vectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 4 \\ 5 \\ h \end{bmatrix}$ are linearly independent.

4. a) (5 pts) True or False: If \mathbf{v}_4 is in the $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is a linearly **independent set**. *Justify your answer.*

b) (5 pts) True or False. If A is a 5×6 matrix, then the columns of A are linearly *dependent*. **Justify your answer.**

c) (5 pts) True or False: If 3×3 matrices A and B each have 3 pivot positions, then A is row equivalent to B . **Justify your answer.**

5. a) (5 pts) Complete the definition: A transformation $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is **linear** if

b) (5 pts) Determine whether the transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ by $T \left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 + 3 \\ 4x_2 \end{bmatrix}$ is a linear transformation.

c) (5 pts) Assume that a linear transformation $T : \mathbb{R}^n \rightarrow \mathbb{R}^n$ (same n) is **not onto**. Carefully prove that T is **not one-to-one**.

Name: _____

Problem	Points	Score
1	25	
2	15	
3	25	
4	15	
5	15	
Total	95	

Math 204: Test 1, Part 2: Maple Application

You may do this section of the test with a partner. Submit only one copy. **Due Wednesday at the start of class.**

6. Read pages 97–99 in Section 1.10 of your text. Pay special attention to Example 3. (You may want to try Exercise 13 on page 101. The answer is the back of the text.) **Note:** Matrix-vector multiplication in Maple is indicated by using a period. So typing $A \cdot x$ in Maple produces the product Ax .

- a) Use the migration matrix in Example 3 BUT assume that the initial population in 2000 was

$$\mathbf{x}_0 = \begin{bmatrix} 700000 \\ 300000 \end{bmatrix}.$$

Compute the population vectors \mathbf{x}_k for $k = 1, \dots, 25$. This will involve 25 separate matrix-vector multiplications but you should only have to type in M and \mathbf{x}_0 and then use Maple to do the rest (tedious). Discuss what you find. How is the population distribution changing? Once you have described what happens, go back to the equation where \mathbf{x}_1 is defined and put a colon $:$ at the end of the line. Hit return and notice that the output disappears. Do this for all other \mathbf{x}_i except \mathbf{x}_{10} , \mathbf{x}_{20} , and \mathbf{x}_{25} . This will save space when you print out your work.

- b) Again use the migration matrix M in Example 3 but now assume that the initial population in 2000 was

$$\mathbf{x}_0 = \begin{bmatrix} 300000 \\ 700000 \end{bmatrix}.$$

Carry out the same work as above. **Time saver:** Copy the work you did in part (a) and then paste it into your document. Change the entries in \mathbf{x}_0 as required. Now look at the command icons at the very top of the window. Click on **!!!**. That will execute all the commands in your document again and so will execute the new material you just pasted in.

- c) **Small Extra Credit.** Find a formula for the vector \mathbf{x}_2 using only the symbols for the migration matrix M and \mathbf{x}_0 . Do the same for \mathbf{x}_5 .
- d) **Small Extra Credit.** Use Maple Help to read about **for** loops or look online. See if you can write a single loop (one command) to do all 25 calculations. This is why Maple can be useful.
7. Select the entire document and put it in 10pt font to save paper by using the font size button at the top of the window. Put your names on top. It should take less than two pages for parts (a)–(c).