1. (12 pts) Let $A = \begin{vmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{vmatrix}$. If $\det A = 3$, evaluate each of the following determinants

(*a*) $\begin{vmatrix} e & f & g & h \\ i & j & k & l \\ a & b & c & d \\ m & n & o & p \end{vmatrix} =$

REASONING/JUSTIFICATION:

(*b*) $\begin{vmatrix} a+i & b+j & c+k & d+l \\ e-i & f-j & g-k & h-l \\ i & j & k & l \\ m & n & o & p \end{vmatrix} =$

REASONING/JUSTIFICATION:

(*c*) $\begin{vmatrix} a & b & c & d \\ e & f & g & h \\ 2i & 2j & 2k & 2l \\ m & n & o & p \end{vmatrix} =$

REASONING/JUSTIFICATION:

(*d*) $\det(2A) =$. Think about part (c) and carefully explain your answer.

2. (*a*) (6 pts) What value(s) of k make the matrix $A = \begin{bmatrix} k & k & 9 \\ 2 & 0 & 0 \\ 3 & 4 & k \end{bmatrix}$ **non-invertible**? Justify your answer; show your work.

(*b*) (6 pts) Prove: If A is invertible, then $A \sim A^T$.

3. (6 pts) Complete the following definitions.

(*a*) An $n \times n$ matrix E is an **elementary matrix** if

(*b*) A $n \times n$ matrix A is **invertible** if

4. (*a*) (8 pts) Find the inverse of $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ 1 & 3 & 1 \end{bmatrix}$ if it exists.

(*b*) (7 pts) Evaluate $\begin{vmatrix} 1 & 2 & 0 & 3 \\ -1 & 1 & -2 & 2 \\ 2 & 4 & 0 & 7 \\ 1 & 0 & 0 & 1 \end{vmatrix}$.

5. (*a*) (5 pts) True or false: If A and B are $n \times n$, then $(A - B)(A + B) = A^2 - B^2$. Justify your answer.

(*b*) (6 pts) Prove: If A is $n \times n$ and the columns of A are independent, then the columns of A^2 span $\mathbb{R}^n$

6. (*a*) (6 pts) True or false: Suppose that $T : \mathbb{R}^n \to \mathbb{R}^n$ is a linear transformation with standard matrix A. If $T(\mathbf{u}) = T(\mathbf{v})$, where $\mathbf{u}$ is not equal to $\mathbf{v}$, then the rows of A^T span $\mathbb{R}^n$. (Carefully prove your answer.)

(*b*) (6 pts) We say that a square matrix B is **symmetric** if $B = B^T$. Prove: Let A be any matrix. If $B = AA^T$, then B is symmetric.

7. (6 pts) Complete the definitions:

(*a*) A transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is **onto** if

(*b*) A transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is **linear** if

(*c*) (6 pts) Determine whether the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 3 \\ 4x_2 \end{bmatrix}$ is a linear transformation.

(*d*) (6 pts) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation, where $m > n$. Carefully prove that T is not onto.

8. (6 pts) Suppose the matrix M reduces to the 3×3 identity matrix through the following series of elementary row operations: (a) $R_1 \leftrightarrow R_2$; (b) $R_2 + 3R_1 \to R_2$; (c) $\frac{1}{2}R_2 \to R_2$; (d) $R_3 - 5R_2 \to R_3$; (e) $R_1 + 2R_2 \to R_1$. Find $\det(M)$.

9. Assume that $T : \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation. Let $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and assume that $T(\mathbf{u}) = T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$ and $T(\mathbf{v}) = T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 6 \\ -1 \end{bmatrix}$.

(*a*) (2 pts) Determine $T(2\mathbf{u} - \mathbf{v})$

(*b*) (10 pts) Determine the standard matrix A for T. You will need to compute T of some special vectors first.

(*c*) (4 pts) Is $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$ in the range of T? Justify your answer.

(*d*) (4 pts) Is T one-to-one? Explain carefully citing appropriate theorems.

You substitution: The following problem may be substituted for any other problem part on the exam. However, this problem is worth at most 5 points. If you substitute, clearly state for which problem this is a substitution (both here and earlier where the substitution takes place).

10. (5 pts) Suppose that T is a linear transformation and that $\{\mathbf{u}, \mathbf{v}\}$ are linearly *dependent* vectors in the domain of T. Prove that $T(\mathbf{u})$ and $T(\mathbf{v})$ are also linearly dependent.

Problem	Points	Score
1	12	
2	12	
3	6	
4	15	
5	11	
6	12	
7	18	
8	6	
9	20	
10	opt 5	
Total	112	

Name: ____________________________________