Directions: Answer all the following questions. Try to phrase your answers in complete sentences so the reader has a narrative instead of just short, clipped phrases or just mathematics symbols. No credit will be given for illegible or disorganized answers.

First some more warm-up exercises about vectors.

1) Graph on a single 2-D axis system all the column vectors
\[ x \begin{bmatrix} 1 \\ -1 \end{bmatrix} \]
for all choices of real number \( x \).

2) Graph on a single 3-D axis system all the column vectors
\[ x \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \]
for all choices of real number \( x \).

3) Prove that any column vector \( \begin{bmatrix} a \\ b \end{bmatrix} \) in the usual x-y plane can be written as a vector sum of the form
\[ x \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y \begin{bmatrix} 0 \\ 1 \end{bmatrix} \]
for suitable choices of \( x \) and \( y \). Yes, this is that easy!

4) Prove that any column vector \( \begin{bmatrix} a \\ b \end{bmatrix} \) in the usual x-y plane can be written as a vector sum of the form
\[ x \begin{bmatrix} 1 \\ 1 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} \]
for suitable choices of \( x \) and \( y \).

5) Prove that there are column vectors \( \begin{bmatrix} a \\ b \end{bmatrix} \) in the plane which cannot be written as the vector sum of the form
\[ x \begin{bmatrix} -2 \\ 4 \end{bmatrix} + y \begin{bmatrix} 1 \\ -2 \end{bmatrix} \]
for any choices of \( x \) and \( y \). Describe in words and draw precisely all those vectors which can be written as such a sum.

6) (Harder) Work problem 22 on page 11 in our text.

Puzzler: Three travelers check into a hotel. The desk clerk tells them the cost of one room is 30 dollars. They each pay with a 10 dollar bill and go to their room. There is a knock on the door. Opening the door, they find a bell hop who tells them that the clerk was reading the wrong chart and the room is only 25 dollars and hands the surprised travelers five dollar bills as a refund. Happy with the reduced room fare, the travelers give the bell hop two dollars as a tip and each keeps one dollar. Suddenly, one of the travelers says, ”Hey, something here does not add up. We
each initially paid 10 dollars for the room, which is a total of 30. But we each got one dollar back so actually we each paid only $9 for the room. Three times nine dollars is 27 plus the 2 dollars we gave the bell hop as a tip makes only 29 dollars. Where is the missing dollar?

Well, where is the missing dollar?