Solutions Compared to Answers.

Notice that I am asking for solutions, not answers. The difference is that solutions include the methods by which you reach an answer or conclusion, not just the answer itself. Be sure that you sufficiently explain all of your results. Give step-by-step justifications, and include the reasoning process that leads you to your conclusions. At this early point in the course, the justifications typically involve definitions of terms or may involve working out some equation. Keep the writing suggestions from Chapter 0 in mind when composing your solution set.

1. Let \( A = \{a, b\} \). Determine \( \mathcal{P}(A) \) and \( |\mathcal{P}(A)| \).

   **SOLUTION.** By definition \( \mathcal{P}(A) \) is the set of all subsets of \( A \). Consequently, in this case \( \mathcal{P}(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\} \). By definition \( |S| \) is the cardinality of \( S \), that is, the number of elements of \( S \). So \( |\mathcal{P}(A)| = 4 \).

2. Give an example of three sets \( A, B, \) and \( C \) so that \( A \in B \), \( B \subset C \), and \( A \not\subseteq C \).

   **SOLUTION.** (After having figured out an answer first.) Let \( A = \{a, b\} \). Since \( A \in B \), we must include the set \( A \) as one of the elements of \( B \). So let \( B = \{\{a, b\}, x, y\} \). Next we need \( B \subset C \), so by definition of proper subset every element of \( B \) must be an element of \( C \) and \( C \) must contain at least one element not in \( B \). Additionally, \( A \) is NOT a subset of \( C \), so at least one element of \( A \) is not in \( C \). So let \( C = \{\{a, b\}, x, y, z\} \). Every element of \( B \) is an element of \( C \) and \( z \in C \) but \( z \notin B \), so \( B \subset C \). Finally, \( a \in A \), but \( a \notin C \), so \( A \not\subseteq C \).

Notice how the definitions were used to justify steps in each of these solutions.