Math 110: Practice for Test 1

1. a) There are 7 continents. If there are 25 ambassadors from different countries visiting the White House, what is the largest number that we can be certain must come from a single continent? Explain your answer.

b) What is the smallest number of ambassadors needed to ensure that there AT LEAST eight from one of the continents? Explain your answer.

2. a) The Gabonacci sequence $G_n$ starts out as $G_1 = 3$ and $G_2 = 2$ and subsequent terms are sums of the previous two terms. What is $G_{12}$? Show your work.

b) The Difonacci sequence $D_n$ starts out as $D_1 = 1$ and $D_2 = 1$ and subsequent terms are sums are given by the difference of the previous two terms: $D_{n+1} = D_n - D_{n-1}$ of the previous two terms. Write out the first 15 terms of the sequence. BE CAREFUL! What do you notice? What would $D_{600}$ be? Explain.

<p>| | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_1$</td>
<td>$D_2$</td>
<td>$D_3$</td>
<td>$D_4$</td>
<td>$D_5$</td>
<td>$D_6$</td>
<td>$D_7$</td>
<td>$D_8$</td>
<td>$D_9$</td>
<td>$D_{10}$</td>
</tr>
</tbody>
</table>

c) What is the Golden Ratio (exact value and decimal approximation)? What is its symbol? Briefly describe the process we used with Fibonacci numbers to determine the Golden Ratio.

3. a) Suppose you start a game of Fibonacci nim with 251 sticks. What is your first move as player 1? Explain your strategy. If Player 2 plays the same number of sticks as you did, what would your next move be?

b) Suppose you are playing Fibonacci nim with another Math 110 student. You are Player 1. You start with 233 sticks. How much should you bet on the game? Explain.

4. a) Use your Sieve of Eratosthenes (just handed back) to find all the prime numbers between 90 and 110. Are there any twin primes in this interval?

b) What does the Goldbach conjecture say you should be able to do with the number 100? Can you do it? (Look at your sieve.)

c) Briefly explain what the first two passes through the Sieve of Eratosthenes do.

d) What does the infinitude of primes mean?

e) What does the Prime Factorization Theorem say?

f) Is 1 a prime? Explain.

5. Your uncle started college in the month of August. He took time off and went into the army and then worked for several additional years before finishing his degree. His graduation was 275 months after he started. What month was his graduation in? Briefly explain your answer expressing your work in modular arithmetic.

6. UPC’s have 12 digits, Call the twelve digits $d_1, d_2, \ldots, d_{11}, c$. The last digit $c$ is the check digit. How is $c$ computed from the other 11 digits.

a) What is the check $c$ digit for the UPC 7-42676-44017-c. (Answer: $c = 8$.)

b) The following is the UPC for a box of Kashi Stoneground 7 Grain Crackers: 0-18d27-61006-9. What is the missing digit? Show your work and explain your answer.

c) Change two digits (that have different values) so that following UPC remains valid. 6-59846-60125-6.

7. Determine the following equivalents.

a) $59 \times 24 \equiv \text{(mod 9)}$. Show your work.

b) Is $7^2 + (5 \times 57) \equiv 40 \pmod{48}$? (The answer is on page 89 in your text.)

c) Is $2^4 + 5^{301} + (6 \times 31) \equiv 3 \pmod{5}$? (The answer is on page 89 in your text.)

d) Is $9^{2000} \equiv 1 \pmod{80}$? (Hint: Write 2000 as $2 \times 1000$.) (The answer is on page 89 in your text.)

e) $5^{500} \pmod{24}$. Show your work. Hint: Write 500 as $2 \times 250$.

f) $5^{501} \pmod{24}$. Show your work.

g) $6^{501} \pmod{36}$. Show your work.
8. a) Solve for \( w \) in the equation below. Show your work.

\[
w = 3 + \frac{2}{3 + \frac{2}{3 + \frac{2}{3 + \frac{2}{\cdots}}}}
\]

b) Solve for \( x \) in the equation below. How can you use part (a)?

\[
= \frac{2}{3 + \frac{2}{3 + \frac{2}{3 + \frac{2}{\cdots}}}}
\]

9. a) Suppose \( m \) and \( n \) are natural numbers. What does the Division Algorithm say about these numbers?

b) Consider the number \( m = (2 \times 3 \times 4 \times 5 \times 6) + 1 \). Give a brief explanation of why \( m \) has no factors smaller than 7 (other than 1).

c) Evaluate the above number \( m \pmod{3} \). Quickly calculate \( m \pmod{2} \), \( m \pmod{4} \), \( m \pmod{5} \), and \( m \pmod{6} \). How are your answers related to the previous part?

d) When we were proving that there were an infinite number of primes, we had to create a number \( m \) such that it was not divisible by 2, 3, 4, 5, ..., \( k \), where \( k \) was some large but unknown integer. How can you create such a number?

e) Consider the number \( m = (2 \times 3 \times 4 \times 5 \times 6) + 2 \). Without working out the value of the number, can you explain why it is NOT prime?

10. We proved the primes were infinite. Are the non-prime natural numbers infinite? Create a list using some pattern if they are.

11. What is a conjecture?

a) What is the Goldbach conjecture? Apply it to \( n = 30 \).

b) What are twin primes? Give three examples. What is the twin prime conjecture?

12. This is a challenging problem to get you to think about the Division Algorithm.

a) A number \( m \) is divided by 20 and has a remainder of 17. Express \( m \) using the division algorithm, \( m = nq + r \). You know the values of three of these numbers. Leave the other as a letter.

b) Take this same number \( m \) and add 184 to it. What is the remainder when this new number is divided by 5? Hint: Use your answer to (a) to help with this.

c) Using the same \( m \), what is \( m + 184 \pmod{5} \)?

13. a) True or false: If \( n \) is a factor of \( m \), then \( m \equiv 0 \pmod{n} \). Explain your answer.

b) Is \( 84 \equiv 36 \pmod{24} \)? Explain.

14. Graduation Day at HWS is always on a Sunday. The seniors celebrate on the day that is 100 days before graduation. What day of the week is that? Justify your answer.

15. Review your journal questions. A couple will be on the exam.