This homework covers sections 1.1 and 1.2. It is due in class Wednesday, January 31.

When writing your solutions, be sure to always show your work and explain your reasoning. Unsupported answers will receive little or no credit. Write your answers carefully — treat homework problems like writing assignments where the goal is to clearly present the solution, not to document the process you went through to arrive at the solution. Use full sentences and paragraphs when appropriate, and consider a process of draft and revision. Typing your solutions is recommended to make it easier to revise, but isn't required, especially in cases where symbols or diagrams can make typing time-consuming.

While you may discuss problems with other students, you should always make the first attempt on a problem yourself and **you must write up your own solutions in your own words**. *You may not collaboratively write solutions or copy a solution that one person in the group writes up.*

1. Construct truth tables to prove the following logical equivalences. Justify your answers by explaining what it is about the truth tables that proves equivalence.

   Note: part (c) is a useful rule that you will need to use in later problems.

   (a) $p \land (q \lor \neg p) \equiv p \land q$

   (b) $p \to (q \to r) \equiv (p \land q) \to r$

   (c) $p \to q \equiv \neg p \lor q$

2. Consider an ordinary deck of 52 playing cards. In such a deck, there are four suits (hearts, diamonds, spades, clubs) and within each suit there are 13 cards (the numbers 2 through 10, jack, queen, king, ace).

   Let h refer to the proposition "this card is a heart" and q refer to the proposition "this card is a queen". For each of the following propositions, express the same meaning in English and determine the number of cards in the deck for which the proposition is true. Don't forget to justify your answers.

   (a) $h \land q$

   (b) $h \lor q$

   (c) $\neg h$

   (d) $h \to q$

   (e) $h \leftrightarrow q$

   (f) $h \oplus q$

3. Construct a truth table to show that the following proposition is a tautology (and explain what it is about the truth table that shows this), then explain in words why this makes sense.

$$(p \wedge q) \to \neg r \equiv \neg(p \wedge q \wedge r)$$

4. Convert each of the following English statements into propositional logic. You should introduce symbols (such as p, q, d, f, etc) to stand in for the simple propositions. State clearly what each symbol stands for.

   Try to express as much of the meaning of each sentence as possible. If there's anything you can't capture, explain.

   (a) The meal is nutritious but not tasty.
   (b) If Alice is smart and lucky, she will be rich or famous.
   (c) Geneva is a city in the Finger Lakes.

5. Convert the following ambiguous English statement into propositional logic. Identify two possible interpretations and, for each, give the proposition and an unambiguous English statement with the same meaning as the proposition.

   The theme park ride is memorable and thrilling or long.

6. Prove the following logical equivalences using boolean algebra, that is, give a chain of logical equivalences that lead from the left side of the equivalence to the right side, where only a single definition or rule of boolean algebra is applied in each step. Be sure to state which definition or rule is applied in each step.

   (a) $p \wedge (q \vee \neg p) \equiv p \wedge q$
   (b) $p \to (q \to r) \equiv (p \wedge q) \to r$

7. Express the negation of each of the following sentences in natural, unambiguous English. Show how you arrive at your answers by first expressing each sentence in propositional logic.

   (a) Contestants must be at least 18 but no older than 65.
   (b) The wine's flavor is complex but not overwhelming.
   (c) If you study hard, you will be successful.

8. (a) Give the *converse*, *contrapositive*, and *negation* of the following proposition.

$$p \to \neg q$$

   (b) Express in natural English the *converse*, *contrapositive*, and *negation* of the following statement.

   If it is raining, there will be no picnic.