This homework covers sections 4.3-4.4. It is due in class Friday, April 26. Hand in a hardcopy of your solutions.

While you may discuss problems with other students, you should always make the first attempt on a problem yourself and **you must write up your own solutions in your own words**. *You may not collaboratively write solutions or copy a solution that one person in the group writes up.*

1. A context-free grammar for a simplified version of propositional logic is shown.

   $S \longrightarrow A$
   $S \longrightarrow B$
   $A \longrightarrow p$
   $A \longrightarrow q$
   $A \longrightarrow r$
   $A \longrightarrow s$
   $B \longrightarrow \neg F$
   $B \longrightarrow F \wedge F$
   $B \longrightarrow F \vee F$
   $B \longrightarrow F \rightarrow F$
   $F \longrightarrow A$
   $F \longrightarrow (B)$

   (a) Draw a parse tree for the proposition

   $$(r \rightarrow p) \wedge q$$

   using this grammar.

   (b) Draw a parse tree for the proposition

   $$((\neg r) \wedge q) \rightarrow ((p \vee r) \wedge s)$$

   using this grammar.

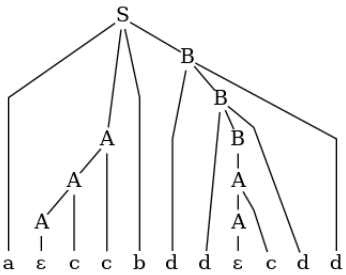
2. The context-free grammar shown generates the language

   $$\{w \in \{a, b, c\}^* \mid \text{every } a \text{ is followed by at least two } b\text{s } \}$$

   Show that this grammar is ambiguous by giving either two parse trees or two left derivations for the string *cabbabbbc*.

   $S \longrightarrow ARS$
   $A \longrightarrow abb$
   $R \longrightarrow bR$
   $R \longrightarrow cR$
   $S \longrightarrow \epsilon$
   $A \longrightarrow \epsilon$
   $R \longrightarrow \epsilon$

3. Consider the parse tree shown. It is based on a context-free grammar G.

   (a) Give the production rules that must be part of G in order for the parse tree to be valid.

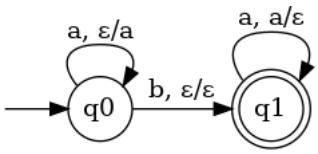
   (b) What is the string being parsed by this parse tree?

   (c) Give the left derivation corresponding to this parse tree.

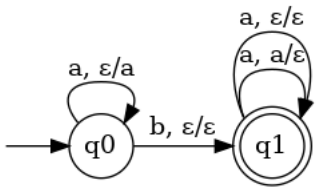
   (d) Give the right derivation corresponding to this parse tree.

4. Give a concise description of the language accepted by each of the following pushdown automata. Explain your answer by describing how the machine works when accepting a string in the language.

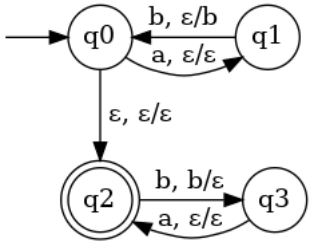
a)



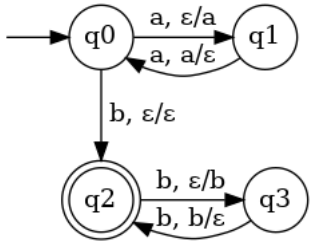
b)



c)



d)



5. For each of the following, draw a transition diagram for a pushdown automaton that accepts the language. Explain how your machines work.

   (a) $L = \{\ a^n b^n \mid n \text{ is a multiple of } 3\ \}$

   (b) Let L be the language over the alphabet $\{(,), [,], a, b\}$ that consists of strings where the parentheses and brackets are properly nested. That is, each opening `(` or `[` is followed by a corresponding `)` or `]`, each closing `)` or `]` is preceded by a corresponding `(` or `]`, and within each matching pair, any other `(`, `[`, `)`, and `]` symbols are also properly nested. For example, $(ab()[bb])$ is properly nested but $(a[b)]a$, $([ba][aba)$, and $ba)$ are not.

6. Show that the language $L = \{\ w \in \{a, b\}^* \mid n_a(w) > n_b(w)\ \}$ is deterministic context-free.