This homework covers sections 4.6 and 5.1. It is due in class Friday, May 3. Hand in a hardcopy of your solutions.

While you may discuss problems with other students, you should always make the first attempt on a problem yourself and **you must write up your own solutions in your own words**. *You may not collaboratively write solutions or copy a solution that one person in the group writes up.*

1. Two grammars for $L = \{\ a^nb^nc^n \mid n \in \mathbb{N}\ \}$ are shown below. (The one on the right was discussed on class.)

$$\begin{aligned} S &\longrightarrow XTZ \\ T &\longrightarrow AbCT \\ T &\longrightarrow \epsilon \\ bA &\longrightarrow Ab \\ CA &\longrightarrow AC \\ Cb &\longrightarrow bC \\ XA &\longrightarrow aX \\ CZ &\longrightarrow Zc \\ X &\longrightarrow \epsilon \\ Z &\longrightarrow \epsilon \end{aligned}$$

$$\begin{aligned} S &\longrightarrow SABC \\ S &\longrightarrow X \\ BA &\longrightarrow AB \\ CA &\longrightarrow AC \\ CB &\longrightarrow BC \\ XA &\longrightarrow aX \\ X &\longrightarrow Y \\ YB &\longrightarrow bY \\ Y &\longrightarrow Z \\ ZC &\longrightarrow cZ \\ Z &\longrightarrow \varepsilon \end{aligned}$$

   (a) Give a derivation for the string $aabbcc$ using the grammar on the left.

   (b) Explain how the grammar on the left works, and compare it to the grammar on the right. (See the 4/19 examples from class for a discussion of how the grammar on the right works.) You can explain how the grammar works by giving comments on the rules — identify what purpose they serve.

2. Two grammars for $L = \{\ ww \mid w \in \{a,b\}^*\ \}$ are shown below. (The one on the right was partially discussed on class.)

$S \longrightarrow HTE$	$S \longrightarrow RT$
$T \longrightarrow aAT$	$R \longrightarrow aRa$
$T \longrightarrow bBT$	$R \longrightarrow bRb$
$Aa \longrightarrow aA$	$R \longrightarrow D$
$Ab \longrightarrow bA$	$D \longrightarrow DP$
$Ba \longrightarrow aB$	$Paa \longrightarrow aPa$
$Bb \longrightarrow bB$	$Pab \longrightarrow bPa$
$AE \longrightarrow Ea$	$Pba \longrightarrow aPb$
$BE \longrightarrow Eb$	$Pbb \longrightarrow bPb$
$Ha \longrightarrow aH$	$PaT \longrightarrow Ta$
$Hb \longrightarrow bH$	$PbT \longrightarrow Tb$
$HE \longrightarrow \epsilon$	$DT \longrightarrow \epsilon$
$T \longrightarrow \epsilon$	

(a) Give a derivation for the string *aabbaabb* using the grammar on the left.

(b) Explain how the grammar on the left works, and compare it to the grammar on the right. (See the 4/22 examples from class for a discussion of how the grammar on the right works.) You can explain how the grammar works by giving comments on the rules — identify what purpose they serve.

3. Create a general grammar for the language $L = \{\ a^n b a^n b a^n \mid n \in \mathbb{N}\ \}$ and explain how the grammar works. You can explain how the grammar works by giving comments on the rules — identify what purpose they serve.

4. Create a general grammar for the language $L = \{\ a^n b^m b c^{nm} \mid n, m \in \mathbb{N}\ \}$ and explain how the grammar works. You can explain how the grammar works by giving comments on the rules — identify what purpose they serve.

Use the Turing Machine web app at https://math.hws.edu/eck/js/turing-machine/TM.html for the following exercises. Be sure to review the instructions at https://math.hws.edu/eck/js/turing-machine/TM-info.html (or click on the link at the top of the simulator page).

Name your Turing machines TM5, TM6, and TM7 according to the problem and fill in the description with a brief description of what the machine does. If you don't do this when you create a new Turing machine, click "Show Edit/Import/Export", then "Grab Current Example". Edit the name and description and click "Apply".

Save the current Turing machine by clicking the "Save File" button. This will create a `.json` *file in your browser's Downloads folder.*

Hand in your solutions electronically by uploading the JSON files to Canvas — look for "HW 11 Turing machine handin" in the Assignments section. Hand in all of your files at once.

5. Create a Turing machine which decides the language

$$L = \{w \in \{a, b\}^* \mid n_a(w) \text{ is a multiple of 3 } \}$$

   The input is a string of as and bs with the machine positioned at the left end of the string. The output of the computation should be 1 if the number of as is a multiple of 3 and 0 otherwise. Note that the *only* thing on the tape at the end should be 0 or 1, and the machine should be positioned on that value.

   Hint: Consider a DFA that accepts this language; the Turing machine will have states that fulfill a similar role.

6. Create a Turing machine which accepts the language

$$L = \{a^n b^m \mid n \geq m \}$$

   The input w is a string of n as followed by m bs with the machine positioned at the left end of the string. The machine should halt if and only if $w \in L$.

   Hint: there are several ways to create a machine that doesn't halt. The simplest is to use `S` ("stay") instead of `L` or `R`, but the machine can also just go left (or right) forever or alternate left and right moves (requires two states).

7. Create a Turing machine which converts unary numbers to binary. The input is a string of as with the machine positioned at the left end of the string. The output of the computation should be the binary number equal to the original number of as. Note that the *only* thing on the tape at the end should be the binary number, and the machine should be positioned at the left end of the number.

   For a complete answer, your machine should also work for empty input — write a 0 in that case.

Hint: Create the binary number to the left of the string of as. Start by writing a 0. Then repeatedly erase the rightmost a, incrementing the binary number each time. Note that adding 1 to a binary number amounts to finding the rightmost 0 in the number, changing it to a 1, then changing all of the 1s to the right of that new 1 to 0. If there is no rightmost 0, add a new 1 on the left and do the same thing (change all of the 1s to the right of that new 1 to 0). For example:

$$
\begin{aligned}
\mathsf{0} &\rightarrow \mathsf{1}\\
\mathsf{1} &\rightarrow \mathsf{10}\\
\mathsf{10} &\rightarrow \mathsf{11}\\
\mathsf{11} &\rightarrow \mathsf{100}\\
\mathsf{100} &\rightarrow \mathsf{101}\\
\mathsf{10111} &\rightarrow \mathsf{11000}
\end{aligned}
$$