This homework covers sections 1.8–1.9 and 2.1. It is due in class Wednesday, February 21. Hand in a hardcopy of your solutions.

For questions 2 and 3, you can use the definitions and facts on pages 52–53 in the book and basic facts from algebra without having to prove them. Two additional algebra facts that may prove useful are the rule for multiplying polynomials:

$$(a+b)(c+d) = ac + bc + ad + bc$$

and a definition for exponents:

$$\begin{aligned} x^0 &= 1 \\ x^k &= x(x^{k-1}) \end{aligned}$$

While you may discuss problems with other students, you should always make the first attempt on a problem yourself and **you must write up your own solutions in your own words**. *You may not collaboratively write solutions or copy a solution that one person in the group writes up.*

1. Write out each of the following sums in full, without using summation notation. Note that this is only asking what the summation notation means, not for you to compute the value of the sum.

   a) $\sum_{j=0}^{5}(2j+1)$ b) $\sum_{n=1}^{6}\frac{1}{2^n}$ c) $\sum_{i=5}^{8}\frac{i-1}{i+1}$

2. Use proof by induction to show that for any integer $n \geq 1$,

$$\sum_{i=1}^{n} i^2 = \frac{n(2n+1)(n+1)}{6}$$

3. Use proof by induction to show that for any integer $n \geq 0$, $3^n - 1$ is divisible by 2.

4. In both cases below, use proof by induction to show that `sum(arr,n)` correctly finds the sum of array elements `arr[0]`, `arr[1]`, ..., `arr[n-1]` for all $n \geq 1$.

   (a)
```
// return the sum of the first n numbers in array arr
int sum ( int[] arr, int n ) {
  if ( n == 1 ) {
    return arr[0];
  } else {
    return sum(arr,n-1) + arr[n-1];
  }
}
```

(b) Use induction with the loop invariant stated.

```
// return the sum of the first n numbers in array arr
int sum ( int[] arr, int n ) {
  int s = arr[0];
  for ( int i = 1 ; i < n ; i++ ) {
    // loop invariant: s is the sum of elements arr[0..i-1] (inclusive)
    s = s+arr[i];
  }
  return s;
}
```

5. Let $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $B = \{3, 6, 9, 12, 15, 18\}$, and $C = \{n \in \mathbb{Z} | -3 \leq n \leq 3\}$. Find the following sets. (You do not need to justify your answers, just state them.)

   a) $A \cup B$ b) $A \cap B$ c) $A \setminus B$ d) $B \setminus A$

   e) C f) $A \cap C$ g) $\mathbb{N} \cup B$ h) $\mathbb{N} \setminus B$

   i) $\mathbb{Z} \setminus A$ j) $\{x \in B \mid x \text{ is even}\}$

6. Let S be the set $S = \{\emptyset, s, \{s\}\}$. Write the power set $\mathcal{P}(S)$. (You do not need to justify your answer.)

7. Disprove by giving a counterexample: For any sets A and B, $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$.

8. Prove using the definitions of $\cap$, $=$, and $\subseteq$: For any sets A and B, if $A \cap B = B$, then $B \subseteq A$.