Finite-State Automata and Regular Languages

Theorem 3.3. *Every language generated by a regular expression can be recognized by an NFA.*

- proof idea
  - definition of regular expression is recursive, so utilize proof by induction
- proof sketch
  - simplest regular expressions are Φ, ε, *a*
    build an NFA for each of these
  - regular expression operators are |, •, *
    build an NFA for each of these

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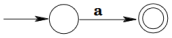
- simplest regular expressions are Φ, ε, *a*

Consider the regular expression Φ. $L(\Phi) = \{\}$. Here is a machine that accepts $\{\}$:

Consider the regular expression ε. $L(\varepsilon) = \{\varepsilon\}$. Here is a machine that accepts $\{\varepsilon\}$:

Consider the regular expression a. $L(a) = \{a\}$. Here is a machine that accepts $\{a\}$:



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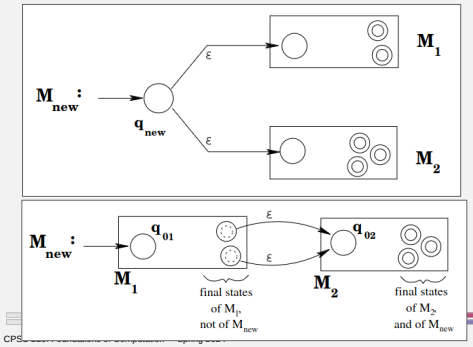


Finite-State Automata and Regular Languages

Complete the proof of Theorem 3.3 by showing how to modify a machine that accepts $L(r)$ into a machine that accepts $L(r^*)$.

Using the construction described in Theorem 3.3, build an NFA that accepts $L((ab \mid a)^*(bb))$.

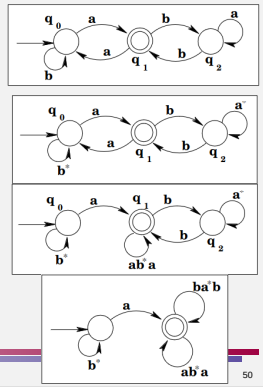
Show that for any DFA or NFA, there is an NFA with exactly one final state that accepts the same language.

Finite-State Automata and Regular Languages

Theorem 3.4. *Every language that is accepted by a DFA or an NFA is generated by a regular expression.*

- a strategy (not a proof)
  - if the DFA/NFA has more than one final state, build an equivalent NFA with a single final state
  - repeatedly
    - replace cycles with self-loops
    - replace simple paths with transitions

    labeling the self-loops/transitions with the string corresponding to the transitions along the cycle/path, dropping no-longer-needed transitions and states
  - read the final result



Finite-State Automata and Regular Languages

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