4. For each of the following languages find a context-free grammar that generates the language:

a) $\{a^n b^m \mid n \geq m > 0\}$
b) $\{a^n b^m \mid n, m \in \mathbb{N}\}$
c) $\{a^n b^m \mid n \geq 0 \wedge m = n + 1\}$
d) $\{a^n b^m c^n \mid n, m \in \mathbb{N}\}$
e) $\{a^n b^m c^k \mid n = m + k\}$
f) $\{a^n b^m \mid n \neq m\}$
g) $\{a^n b^m c^r d^t \mid n + m = r + t\}$
h) $\{a^n b^m c^k \mid n \neq m + k\}$

Context-Free vs Regular

- there are context-free languages which are not regular
  - $L = \{ a^n b^n \mid n \in \mathbb{N} \}$ was shown to be not regular in section 3.7 (theorem 3.7) but it is generated by the context-free grammar shown

$$S \longrightarrow aSb$$
$$S \longrightarrow \varepsilon$$

Context-Free vs Regular

- every regular language is context-free

Definition 4.3. A ***right-regular grammar*** is a context-free grammar in which the right-hand side of every production rule has one of the following forms: the empty string; a string consisting of a single non-terminal symbol; or a string consisting of a single terminal symbol followed by a single non-terminal symbol.

Theorem 4.4. *A language L is regular if and only if there is a right-regular grammar G such that $L = L(G)$. In particular, every regular language is context-free.*

  - idea of proof
    - build an NFA with states corresponding to the non-terminal symbols of the grammar
    - production $A \rightarrow bC$ corresponds to a transition from state A to state C while reading symbol b
    - production $A \rightarrow B$ corresponds to an ε-transition from state A to state B
    - production $A \rightarrow \varepsilon$ corresponds to A being a final state

Context-Free Languages

Theorem 4.3. *Suppose that L and M are context-free languages. Then the languages $L \cup M$, LM, and L^* are also context-free.*

9. Suppose that G and H are context-free grammars. Let $L = L(G)$ and let $M = L(H)$. Explain how to construct a context-free grammar for the language LM. You do not need to give a formal proof that your grammar is correct.

10. Suppose that G is a context-free grammar. Let $L = L(G)$. Explain how to construct a context-free grammar for the language L^*. You do not need to give a formal proof that your grammar is correct.

11. Suppose that L is a context-free language. Prove that L^R is a context-free language. (Hint: Given a context-free grammar G for L, make a new grammar, G^R, by reversing the right-hand side of each of the production rules in G. That is, $A \longrightarrow w$ is a production rule in G if and only if $A \longrightarrow w^R$ is a production rule in G^R.)