General Grammars

Definition 4.6. A ***grammar*** is a 4-tuple (V, Σ, P, S), where:
1. V is a finite set of symbols. The elements of V are the non-terminal symbols of the grammar.
2. Σ is a finite set of symbols such that $V \cap \Sigma = \emptyset$. The elements of Σ are the terminal symbols of the grammar.
3. P is a set of production rules. Each rule is of the form $u \longrightarrow x$ where u and x are strings in $(V \cup \Sigma)^*$ and u contains at least one symbol from V.
4. $S \in V$. S is the start symbol of the grammar.

- a context-free grammar is a grammar where the rules are limited to the form $A \rightarrow x$ (a single non-terminal on the left)

General Grammars

- (general) grammars are more powerful than context-free grammars
  - we've seen several examples
- are there languages that can't be produced by grammars?
  - yes – for an alphabet Σ, there uncountably many languages but only countably many that can be generated by grammars

1. Find a derivation for the string *caabcb*, according to the first example grammar in this section. Find a derivation for the string *aabbcc*, according to the second example grammar in this section. Find a derivation for the string *aaaa*, according to the third example grammar in this section.

$S \longrightarrow SABC$
$S \longrightarrow \varepsilon$
$AB \longrightarrow BA$
$BA \longrightarrow AB$
$AC \longrightarrow CA$
$CA \longrightarrow AC$
$BC \longrightarrow CB$
$CB \longrightarrow BC$
$A \longrightarrow a$
$B \longrightarrow b$
$C \longrightarrow c$

$n_a(w) = n_b(w) = n_c(w)$

$S \longrightarrow SABC$
$S \longrightarrow X$
$BA \longrightarrow AB$
$CA \longrightarrow AC$
$CB \longrightarrow BC$
$XA \longrightarrow aX$
$X \longrightarrow Y$
$YB \longrightarrow bY$
$Y \longrightarrow Z$
$ZC \longrightarrow cZ$
$Z \longrightarrow \varepsilon$

$a^nb^nc^n$

$S \longrightarrow DTE$
$T \longrightarrow BTA$
$T \longrightarrow \varepsilon$
$BA \longrightarrow AaB$
$Ba \longrightarrow aB$
$BE \longrightarrow E$
$DA \longrightarrow D$
$Da \longrightarrow aD$
$DE \longrightarrow \varepsilon$

a^{n^2}